Multi-stage Large-scale Multiobjective Evolutionary Algorithm Integrating Multidirectional Sampling and Branch Search

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***Abstract*—Most existing algorithms for large-scale multiobjective optimization problems (LSMOPs) suffer from monotonous and fixed search strategies and evolutionary stages. Consequently, the solutions generated by these algorithms struggle to achieve rapid convergence toward the Pareto front. To address this issue, this paper proposes a Multi-stage Large-scale Multiobjective Evolutionary Algorithm integrating Multi-directional sampling and Branch Search (LMOEA-MBMS). Specifically, a breadth multi-directional sampling strategy is first introduced to explore five distinct directions correlated with the sample solutions. Subsequently, a depth branch search strategy is implemented to further explore the non-dominated solutions of these samples along the sampling directions, thereby enhancing the algorithm's convergence. Finally, a multi-stage environmental selection framework is designed to achieve the specific goals of the algorithm across different evolutionary phases. The proposed algorithm was evaluated against six state-of-the-art algorithms on eighteen LSMOPs and two practical problems of Time-varying Ratio Error Estimation (TREE). Experimental results demonstrate that the performance of the proposed algorithm outperforms other state-of-the-art methods, effectively improving both convergence and solution diversity, with significant advancements in both theory and application.**

***Index Terms*—Large-scale multiobjective optimization, breadth multi-directional sampling, depth branch search, multi-stage environmental selection.**

I. Introduction

Large-scale multiobjective optimization problems (LSMOPs) typically involve hundreds or thousands of decision variables and require the simultaneous optimization of two or more conflicting objectives. The general problem can be formulated as follows:

$$\begin{aligned} \text{minimize} \quad & \mathbf{F}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_M(\mathbf{x})) \\ \text{subject to} \quad & \mathbf{x} \in \Omega \subseteq \mathbb{R}^D \end{aligned} \tag{1}$$

where $\mathbf{x} = (x_1, \ldots, x_D)$ is the D-dimensional decision vector ($D \geq 100$), and Ω denotes the decision space. $\mathbf{F}(\mathbf{x})$ is the objective vector consisting of M ($M \geq 2$) conflicting objectives, and the objective space is denoted by $\Theta \subseteq \mathbb{R}^M$.

In Eq. (1), the M objectives conflict with each other, meaning no single optimal solution can minimize all objectives simultaneously. Instead, a set of Pareto optimal solutions exists, representing unique trade-offs where no objective can be improved without sacrificing at least one other. The set of all such solutions in the decision space is referred to as the Pareto optimal set (PS), and its mapping in the objective space is known as the Pareto front (PF). The ultimate goal of solving LSMOPs is to identify a set of solutions that converge closely to the PF while maintaining a uniform distribution [1].

Large-scale multiobjective evolutionary algorithms (LSMOEAs) are widely utilized to solve LSMOPs due to their robust global search capabilities. Existing LSMOEAs can be categorized into four primary types based on their technical approaches:

1) *Cooperative Coevolution (CC):* Cao et al. [2] proposed a distributed parallel cooperative multiobjective evolutionary algorithm that optimizes decision variables through grouping and parallel cooperation. Antonio et al. [3] introduced a CC framework focused on the coevolution of decision variable subcomponents.

2) *Problem Reformulation (PR):* Zille et al. [4] presented the Weight Optimization Framework (WOF), which transforms large-scale problems into lower-dimensional versions by optimizing weight vectors. Recently, Xiong et al. [5] developed an algorithm using autoencoders for efficient problem dimensionality transformation.

3) *Decision Variable Analysis (DVA):* Ma et al. [6] designed an algorithm based on DVA to handle large-scale variables by analyzing their underlying properties. He et al. [7] further addressed LSMOPs through a reformulated DVA approach.

4) *Novel Search Strategies:* Li et al. [8] utilized a tensorized NSGA-III for GPU-accelerated many-objective optimization. Liu et al. [9] proposed a competitive swarm optimizer (CSO), while Shi et al. [10] developed a co-competitive PSO based on dimensionality reduction. Additionally, Zhang et al. [11] introduced a large-scale CSO based on regional multidirectional search.

Despite these advancements, most existing algorithms rely on static search strategies or invariant evolutionary stages, which hinders rapid convergence to the PF. To address this limitation, this paper proposes a multi-stage large-scale multiobjective evolutionary algorithm integrating multi-directional sampling and branch search (LMOEA-MBMS). It can enhance the convergence, diversity, and stability of algorithms. Compared to traditional dimensionality reduction methods, the approach proposed in this paper is more competitive.LMOEA-MBMS is compared with state-of-the-art LSMOEAs on a large-scale multiobjective test set and TREE application test set.Experimental results demonstrate that LMOEA-MBMS achieved the best performance across all test problems, proving effective in solving LSMOPs.And the experiment analyzed the effectiveness of each component within LMOEA-MBMS and its scalability for solving problems of varying dimensions. The main contributions of the study are summarized as follows:

- A breadth multi-directional sampling strategy is proposed, which performs multidirectional sampling across five sample-decorrelated directions, thereby enhancing the distribution of the population within the Pareto frontier space.
- A deep branch search strategy is proposed to further explore sample solutions along sampling directions, thereby accelerating the population's convergence toward the Pareto frontier.
- A novel multi-stage environmental selection framework is designed, comprising a global exploration stage, a local exploitation stage, and a collaborative equilibrium search stage, thereby enhancing the algorithm's global exploration capabilities and local exploitation capabilities.

II. Proposed Algorithm

A. Framework of LMOEA-MBMS

The overall framework of the proposed LMOEA-MBMS is illustrated in Fig. 1. The architecture consists of two primary modules: 1) an offspring generation module integrating breadth and depth search via directional sampling, and 2) a multi-stage environmental selection module. As depicted in the conceptual flow, the initial population P_t undergoes expansion in Module 1 to generate the offspring population O_t. Subsequently, O_t is processed by Module 2 to produce the next generation P_{t+1}.

During the evolutionary process, Module 1 expands the search breadth and depth for each individual, increasing the probability of approaching the Pareto front and accelerating convergence. Module 2 then facilitates a multi-stage evolution to balance the algorithm's exploration and exploitation capabilities. In stage 1, Particles absorb guidance information for rapid convergence and global exploration. In stage 2, Learns from elite solutions to balance exploration and local exploitation. In stage 3, Performs fine-grained optimization within the population to maintain solution diversity.

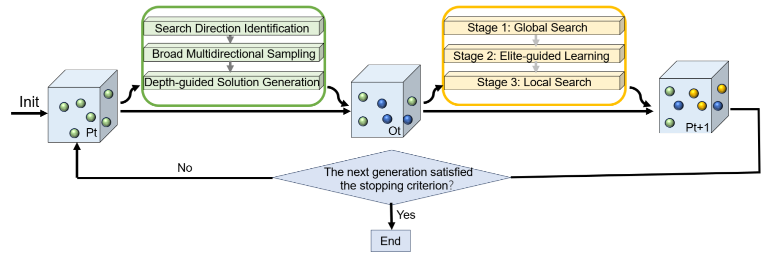

Fig. 1. The framework of LMOEA-MBMS, illustrating the integration of multidirectional sampling (Module 1) and multi-stage environmental selection (Module 2).

B. Breadth Multi-directional Sampling

Metaheuristic algorithms often face challenges due to the complex topology of Pareto fronts in large-scale spaces. To address this, a breadth multi-directional sampling strategy is proposed, extending traditional directional sampling [12]. This strategy enhances sampling coverage, ensuring that sampling directions are more likely to intersect with the Pareto front.

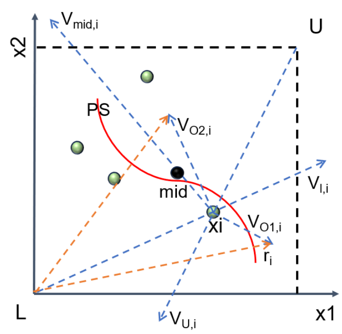

Fig. 2. Illustration of the breadth multi-directional sampling mechanism.

As shown in Fig. 2, let $\mathbf{L}$ and $\mathbf{U}$ represent the lower and upper bounds of the decision space, respectively, and $\mathbf{mid}$ denote the center point.

For particle $\mathbf{x}_i$, five search directions are set: $\mathbf{v}_{l,i}$, $\mathbf{v}_{u,i}$, $\mathbf{v}_{o1,i}$, $\mathbf{v}_{o2,i}$, and $\mathbf{v}_{mid,i}$. Where $\mathbf{v}_{l,i}$ is the direction vector passing through $\mathbf{x}_i$ and originating from the lower bound $\mathbf{L}$. $\mathbf{v}_{l,i}$ can be determined according to Equations (2) .

$$\mathrm{v}_{l,i} = x_i - L \tag{2}$$

$\mathbf{v}_{u,i}$is the direction vector passing through $\mathbf{x}_i$ and originating from the upper bound $\mathbf{U}$.$\mathbf{v}_{u,i}$ can be determined according to Equations (3) .

$$\mathrm{v}_{u,i} = x_i - U \tag{3}$$

$\mathbf{v}_{mid,i}$ is the direction vector passing through $\mathbf{x}_i$ and ending at the midpoint $\mathbf{mid}$.$\mathbf{v}_{mid,i}$ can be determined according to Equations (4) .

$$\mathrm{v}_{mid,i} = mid - x_i \tag{4}$$

$\mathbf{v}_{o1,i}$ and $\mathbf{v}_{o2,i}$ are direction vectors passing through $\mathbf{x}_i$ and perpendicular to $\mathbf{v}_{u,i}$ and $\mathbf{v}_{l,i}$, respectively. Their specific orientations are determined by the random auxiliary vector

$\mathbf{r}_i$. $\mathbf{v}_{o1,i}$ and $\mathbf{v}_{o2,i}$ can be determined according to Equations (5) and (6).

$$\mathrm{v}_{o1,i} = r_1 - proj_{\mathbf{v}_{u,i}}(\mathbf{r}_1) \tag{5}$$

$$\mathrm{v}_{o2,i} = r_2 - proj_{\mathbf{v}_{l,i}}(\mathbf{r}_2) \tag{6}$$

Where $\mathbf{r}_i$ denotes a randomly generated vector, $\mathbf{r}_i \in [-\mathbf{0.5}, \mathbf{0.5}]^D$, and $\mathbf{proj}_{\mathbf{v_{l,i}}}(\mathbf{r_i})$ represents the projection of $\mathbf{r}_i$ onto $\mathbf{v}_{(l,i)}$. $\mathbf{proj}_{\mathbf{v_{l,i}}}(\mathbf{r_i})$ is calculated as:

$$\mathrm{proj}_{\mathbf{v}_{l,i}}(\mathbf{r}_i) = \frac{(\mathbf{r}_i \cdot \mathbf{v}_{l,i})}{\|\mathbf{v}_{l,i}\|^2}\mathbf{v}_{l,i} \tag{7}$$

This strategy enables the algorithm to evolve toward the Pareto frontier as much as possible, thereby accelerating convergence.

C. Guided Solution Generation Integrating Depth Branch Search

In theory, the algorithm can find the solution closest to the Pareto frontier along each search direction.However, when multiple solutions exist in the search direction or no solution exists, it may not be possible to find a guiding solution.Therefore, it was decided to conduct a deep search along the search direction, proposing a guidance solution generation strategy that integrates deep branch search.The advantage of this strategy lies in its ability to continue depth-first search on top of breadth-first sampling, thereby obtaining optimal solutions. This enhances the algorithm's convergence speed and increases the diversity of guided solutions.The strategy as follows:

$$\mathbf{x}_j^{l,i} = \mathbf{L} + s_j^{l,i}\frac{\mathbf{v}_{l,i}}{\|\mathbf{v}_{l,i}\|} + \alpha \cdot d_j^{l,i} \cdot \mathbf{w}_j^{l,i} \tag{8}$$

where α is the depth perturbation intensity coefficient, $\mathbf{w}_j^{l,i}$ is a random perturbation direction vector for the branch search, and $d_j^{l,i}$ represents the depth distance defined as:

$$d_j^{l,i} = \beta \cdot s_j^{l,i} \tag{9}$$

where β is a scaling factor proportional to the base distance.

Proposition 1 (The Multi-directional Sampling and Branch Search collaborative strategy (MB) is effective in solving LSMOPs.)

Proof. First, construct a vector set $\mathbf{V}_i$ composed of linearly independent boundary vectors and orthogonal direction vectors to generate a full-rank search matrix $\mathbf{Rank}D$. Based on computational geometric coverage properties, the linear space spanned by this direction set is equivalent to the high-dimensional decision space $\mathbf{R}^D$. This ensures complete exploration on non-convex/discontinuous PFs while maintaining population diversity.

Second, introduce a deep perturbation term $\alpha(\beta\mathbf{s})\mathbf{w}$ into the search, confining exploration to the local neighborhood of elite solutions.

Finally, based on the principle of fine approximation, this mechanism focuses sampling within the ϵ neighborhood ϵ-$\mathbf{N}(\mathbf{X}_j, \epsilon)$ of elite solutions. This enhances local development capabilities in high-dimensional spaces, significantly improving convergence accuracy for real-world PFs. □

D. Multi-stage Environmental Selection Framework

Multi-stage evolution can provide effective supplementation and guidance for the generation of superior offspring solutions, while the preservation and transmission of information are primarily conducted during environmental selection. This paper proposes a multi-stage environmental selection strategy, as illustrated in Algorithm 1. The input consists of a set of reference vectors W, the initial population size N, and a threshold N_ϵ used to determine the selection method. The output is the parent population P_{t+1} for the next generation.

In the first stage (Global Exploration), within the parent population P_t, particles absorb information from the guiding solution g_k to rapidly approach the Pareto front, followed by adaptive environmental selection (Lines 1-7). In the second stage (Local Exploitation), within the intermediate population P_{t1}, particles learn from the elite solution e_j to enhance convergence performance, followed by adaptive environmental selection (Lines 8-9). In the third stage (Cooperative Balanced Search), within the intermediate population P_{t2}, particles learn by pairing with population P'_{t2} to further improve the diversity and distribution of the solutions, followed by adaptive environmental selection (Lines 10-11).

Algorithm 1 Multi-stage Environmental Selection Algorithm

Input: W (reference vectors), N (initial population size), N_ϵ (threshold)
Output: P_{t+1} (parent population for the next generation)

1: **Stage 1:** $P_{t1} \leftarrow$ Stage 1 Operator(P_t, g_k);
2: Normalize objective values and assign each individual to the nearest reference vector;
3: **if** the number of reference vectors with at least one individual assigned $\geq N_\epsilon$ **then**
4: Execute decomposition-based environmental selection strategy;
5: **else**
6: Execute non-dominated sorting;
7: **end if**
8: **Stage 2:** $P_{t2} \leftarrow$ Stage 2 Operator(P_{t1}, e_j);
9: Repeat environmental selection process (Lines 2-7);
10: **Stage 3:** $P_{t+1} \leftarrow$ Stage 3 Operator(P_{t2}, P'_{t2});
11: Repeat environmental selection process (Lines 2-7);
12: **return** P_{t+1};

III. Experimental Settings

A. Comparison Algorithms and Test Problems

To evaluate the performance of the proposed LMOEA-MBMS, six state-of-the-art LSMOEAs are selected as competing algorithms: LMOEADS [12], IMTCMO_BS [13], MOCGDE [14], IMCMOEAD [15], CCGDE3 [2], and LMOCSO [16].

For performance assessment, eighteen LSMOP test functions and two practical problems related to Time-varying Ratio Error Estimation (TREE) [17] are utilized. The LSMOP test suite ensures scalability and generalizability, while the TREE problem set, which involves complex variable interactions and objective correlations, consists of measured voltage and phasor data from various substations.

The Inverted Generational Distance (IGD) [18] and Hypervolume (HV) [19] metrics are employed to quantitatively evaluate the performance. The Inverted Generational Distance is defined as:

$$IGD(P,S)=\frac{\sum_{s\in S}\min_{\mathbf{x}\in P}\|\mathbf{F}(\mathbf{x})-s\|_2}{|S|} \tag{10}$$

where P is the set of objective vectors obtained by the algorithm, and S is a set of uniformly distributed points sampled from the true PF. A smaller IGD value indicates better convergence and diversity. The Hypervolume metric is defined as:

$$HV(P,\mathbf{r})=\Lambda\left(\bigcup_{\mathbf{z}\in P}[\mathbf{z},\mathbf{r}]\right) \tag{11}$$

In (11), $\Lambda(\cdot)$ denotes the Lebesgue measure, $\mathbf{r}$ is the reference point, and $[\mathbf{z},\mathbf{r}]$ represents the hyperrectangle formed by the solution $\mathbf{z}$ and the reference point. A larger HV value indicates superior overall performance.

B. Experimental Environment and Parameter Settings

In our experiments, the number of objectives M is set to 3. To verify the algorithm's actual performance and scalability, the number of decision variables D is configured at 300, 500, 600, or 1000. Each algorithm is executed 30 times independently for each test problem to eliminate evolutionary randomness. The maximum number of evaluations is set to 10,000. All simulations are implemented using Matlab R2021b within the PlatEMO platform [20].

IV. Results And Analysis

A. Analysis of the Ablation Experiment Results

An ablation study is first conducted to evaluate the individual contributions of the proposed components. The integration of breadth multi-directional sampling and depth-guided search is denoted as the MB module, while the multi-stage environmental selection strategy is denoted as the MS module. To demonstrate the importance of these modules within LMOEA-MBMS, the performance of LMOEA-MS (without MB) and LMOEA-MB (without MS) is compared with the complete LMOEA-MBMS. The quantitative results and convergence curves are summarized in Table I and Fig. 3, respectively.

Statistical analysis using the Wilcoxon rank-sum test ($\alpha = 0.05$) indicates that LMOEA-MB significantly outperforms the baseline algorithm in terms of the IGD metric, confirming the critical role of the MB module in accelerating convergence. Meanwhile, LMOEA-MS performs effectively on problems with strong variable correlations and complex local optima, such as LSMOP8, which highlights the importance of the MS module in enhancing solution diversity and exploration. The results demonstrate a cooperative effect: the MS module manages early-stage exploration, while the MB module ensures precise convergence in the later stages, enabling balanced optimization across various problem types.

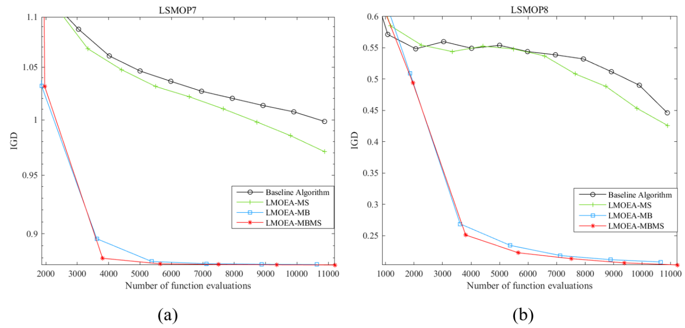

Fig. 3. Convergence curves of the ablation study on LSMOP7 and LSMOP8.

B. Comparison with State-of-the-Art Algorithms

The performance of LMOEA-MBMS is evaluated against six state-of-the-art algorithms on the LSMOP benchmark suite. As shown in Table II,'+‘, '-', and '=' denote that the comparison algorithm's results are significantly better than LMOEA-MBMS, significantly worse than LMOEA-MBMS, and statistically similar to LMOEA-MBMS, respectively. The best result is indicated in bold. The experimental results demonstrate that LMOEA-MBMS possesses significant performance advantages in handling these problems. Specifically, the proposed algorithm achieves the best results in 16 out of 18 test instances. The proportions of problems where LMOEA-MBMS significantly outperforms LMOEADS, IMTCMO_BS, MOCGDE, IMCMOEAD, CCGDE3, and LMOCSO are 15/18, 18/18, 18/18, 18/18, 18/18, and 18/18, respectively.

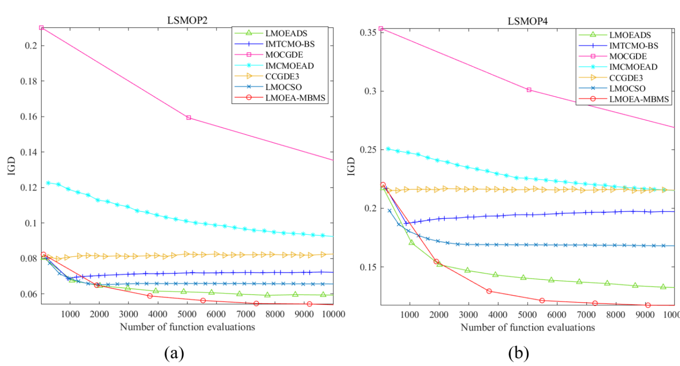

Fig. 4. Convergence curves of LMOEA-MBMS and competing algorithms on representative LSMOP problems.

The scalability of the algorithm is demonstrated by extending the decision variables from 500 to 1000 dimensions, illustrating that LMOEA-MBMS remains effective for large-scale optimization. As shown in Fig. 4, LMOEA-MBMS maintains stable and efficient convergence across different stages. In the early stage of evolution, the algorithm prevents premature

TABLE I

IGD Results (Standard Deviation) of LMOEA-MBMS and Comparison Algorithms in Ablation Study

Problem	Dec.	Baseline Algorithm	LMOEA-MS	LMOEA-MB	LMOEA-MBMS
LSMOP8	500	4.4612e-1 (8.57e-2) −	4.2600e-1 (9.31e-2) −	2.0792e-1 (8.60e-3) −	**2.0311e-1 (5.49e-3)**
	1000	4.8375e-1 (8.35e-2) −	4.1700e-1 (8.53e-2) −	1.9312e-1 (5.10e-3) =	**1.9209e-1 (5.80e-3)**
LSMOP9	500	9.5048e-1 (4.19e-1) −	7.3432e-1 (2.96e-1) −	6.5827e-1 (1.87e-1) =	**6.0011e-1 (2.64e-2)**
	1000	9.9947e-1 (4.47e-1) −	1.0040e+0 (4.60e-1) −	5.9332e-1 (1.52e-2) −	**5.9031e-1 (6.54e-3)**
+/-/=		0/4/0	0/4/0	0/2/2	

TABLE II

IGD Values (Standard Deviation) of LMOEA-MBMS and Competing Algorithms on LSMOP Problems

Problem	Dec.	LMOEADS	IMTCMO_BS	MOCGDE	IMCMOEAD	CCGDE3	LMOCSO	LMOEA-MBMS
LSMOP1	500	7.6142e-1 (3.60e-2) −	2.2262e+0 (1.77e+0) −	1.2079e+1 (1.22e+0) −	9.3584e+0 (4.94e-1) −	1.0357e+1 (1.09e+0) −	2.0888e+0 (1.61e-1) −	**4 .6885e-1 (1.56e-2)**
	1000	7.6947e-1 (2.23e-2) −	2.6908e+0 (1.99e+0) −	1.3181e+1 (1.35e+0) −	1.0760e+1 (4.99e-1) −	1.0406e+1 (1.46e+0) −	2.1838e+0 (1.52e-1) −	**4.6370e-1 (1.14e-2)**
LSMOP2	500	5.9146e-2 (1.84e-3) −	7.2045e-2 (3.68e-4) −	1.3484e-1 (2.19e-2) −	9.2304e-2 (4.56e-3) −	8.2526e-2 (2.95e-3) −	6.5574e-2 (8.29e-4) −	**5.3356e-2 (1.64e-3)**
	1000	4.8596e-2 (1.74e-3) −	5.4173e-2 (3.46e-4) −	1.5585e-1 (3.60e-2) −	8.7242e-2 (6.81e-3) −	6.6632e-2 (3.09e-3) −	5.2465e-2 (2.63e-4) −	**4.6757e-2 (1.79e-3)**
LSMOP3	500	8.6171e-1 (2.16e-3) =	3.3453e+1 (4.04e+1) −	5.0429e+1 (3.92e+1) −	3.2708e+2 (3.50e+2) −	2.5146e+1 (4.37e+0) −	1.8006e+1 (3.68e+0) −	**8.6036e-1 (7.67e-4)**
	1000	8.6227e-1 (3.62e-3) −	3.5064e+1 (4.27e+1)−	6.3536e+2 (1.42e+3) −	6.5084e+2 (1.03e+3) −	2.4023e+1 (4.75e+0) −	2.1123e+1 (1.68e+1) −	**8.6029e-1 (7.23e-4)**
LSMOP4	500	1.3195e-1 (2.94e-3) −	1.9715e-1 (2.03e-2) −	2.6826e-1 (3.69e-2) −	2.1506e-1 (3.60e-3) −	2.1534e-1 (5.99e-3) −	1.6800e-1 (1.81e-3) −	**1.1709e-1 (1.82e-3)**
	1000	8.4902e-2 (1.99e-3) −	1.2378e-1 (6.17e-3) −	2.3161e-1 (3.62e-2) −	1.4667e-1 (8.15e-3) −	1.3226e-1 (4.13e-3) −	1.0558e-1 (8.73e-4) −	**7.8518e-2 (1.76e-3)**
LSMOP5	500	6.7829e-1 (1.35e-1) −	7.1852e+0 (5.66e+0) −	1.9855e+1 (3.06e+0) −	1.6841e+1 (2.00e+0) −	1.7096e+1 (1.42e+0) −	4.7803e+0 (4.94e-1) −	**5.2449e-1 (6.24e-3)**
	1000	7.5328e-1 (1.62e-1) −	7.6587e+0 (6.01e+0) −	2.1646e+1 (1.32e+0) −	1.9233e+1 (1.35e+0) −	1.8318e+1 (1.72e+0) −	4.8885e+0 (3.32e-1) −	**5.2674e-1 (7.12e-3)**
LSMOP6	500	**8.1007e-1 (3.56e-2) +**	1.8134e+3 (1.77e+3) −	1.6232e+4 (1.35e+4) −	2.2974e+4 (6.36e+3) −	2.2332e+4 (7.98e+3) −	2.3220e+3 (1.68e+3) −	1.2948e+0 (3.50e-4)
	1000	**8.1336e-1 (7.04e-3) +**	1.6212e+3 (1.93e+3) −	3.2342e+4 (2.71e+4) −	3.2597e+4 (6.51e+3) −	2.7503e+4 (4.92e+3) −	3.3114e+3 (2.25e+3) −	1.3145e+0 (5.24e-4)
LSMOP7	500	1.0134e+0 (1.77e-2) −	1.8350e+3 (4.94e+3) −	1.5947e+3 (4.84e+3) −	3.4243e+3 (3.61e+3) −	1.3086e+0 (1.22e-2) −	1.2188e+0 (4.32e-2) −	**8.7421e-1 (5.55e-4)**
	1000	9.5340e-1 (2.97e-2) −	5.0770e+3 (9.06e+3) −	4.6349e+3 (7.47e+3) −	5.6711e+3 (4.33e+3) −	1.1056e+0 (4.74e-3) −	1.3656e+1 (6.89e+1) −	**8.5355e-1 (2.85e-4)**
LSMOP8	500	4.6588e-1 (9.39e-2) −	6.6152e-1 (3.15e-1) −	3.2992e+0 (4.69e+0) −	9.7916e-1 (4.39e-1) −	9.5808e-1 (3.56e-2) −	6.6570e-1 (1.04e-1) −	**2.0673e-1 (5.91e-3)**
	1000	4.8375e-1 (8.35e-2) −	6.3800e-1 (6.93e-1) −	2.0914e+0 (2.96e+0) −	1.3746e+0 (9.28e-1) −	9.5270e-1 (2.87e-2) −	6.6303e-1 (1.04e-1) −	**1.9209e-1 (5.80e-3)**
LSMOP9	500	7.9746e-1 (3.41e-1) −	1.9275e+1 (1.99e+1) −	8.1812e+1 (3.97e+1) −	1.0819e+2 (6.92e+0) −	7.9914e+1 (7.48e+0) −	8.0418e+0 (3.28e+0) −	**6.0871e-1 (6.84e-2)**
	1000	9.9947e-1 (4.47e-1) −	3.2714e+1 (2.48e+1) −	1.4572e+2 (7.65e+0) −	1.3140e+2 (7.38e+0) −	8.9548e+1 (9.02e+0) −	2.3677e+1 (7.23e+0) −	**5.9031e-1 (6.54e-3)**
+/-/=		2/15/1	0/18/0	0/18/0	0/18/0	0/18/0	0/18/0	

TABLE III

HV Values (Standard Deviation) of LMOEA-MBMS and Competing Algorithms on TREE Problems

Prob.	D	LMOEADS	IMTCMO_BS	MOCGDE	IMCMOEAD	CCGDE3	LMOCSO	LMOEA-MBMS
TREE4	600	9.4272e-1 (4.30e-3) −	9.4766e-1 (3.12e-3) −	NaN (NaN)	NaN (NaN)	NaN (NaN)	NaN (NaN)	**9.4953e-1 (9.22e-4)**
TREE5	600	9.2700e-1 (8.72e-4) =	9.2319e-1 (7.72e-3) −	NaN (NaN)	NaN (NaN)	NaN (NaN)	NaN (NaN)	**9.2801e-1 (5.45e-4)**
+/-/=		0/1/1	0/2/0	0/0/0	0/0/0	0/0/0	0/0/0	

convergence to local optima and increases solution diversity by maintaining a moderate speed. In the later stages, it accelerates convergence to improve the final accuracy. Overall, LMOEA-MBMS exhibits a superior balance between diversity and convergence.

C. Performance on TREE Problems

The performance of LMOEA-MBMS and its comparison algorithms was evaluated on real TREE problems, as shown in Table III. LMOEA-MBMS outperformed all comparison algorithms.

V. Conclusion

To improve the performance of evolutionary algorithms in solving LSMOPs, this paper proposes a multi-stage large-scale multiobjective evolutionary algorithm integrating multi-directional sampling and branch search (LMOEA-MBMS). This method maintains an effective and reliable balance between convergence and diversity. By combining a breadth multi-directional sampling strategy with a depth branch search strategy, the algorithm leverages the respective advantages of breadth and depth search to explore high-dimensional decision spaces effectively. The generated guided solutions are then incorporated into a three-stage environmental selection framework. This framework first performs global exploration to avoid premature convergence, followed by intensive exploitation in high-quality regions, and finally balances convergence with solution diversity. Experimental results confirm that the proposed algorithm outperforms six state-of-the-art methods across various problems. Specifically, it achieves superior performance on test suites involving three objectives and thousands of decision variables. Future work will focus on further refining the algorithmic modules and applying the proposed method to additional practical optimization problems.

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