A Parameter-Free Joint-Space Metric for Multimodal Multiobjective Optimization

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***Abstract*—The evaluation of multimodal multi-objective evolutionary algorithms (MMEAs) is crucial for effective algorithm development and selection, yet current performance metrics often assess convergence and diversity in isolation within either the objective space or the decision space. Metrics such as Inverted Generational Distance (IGD) and Inverted Generational Distance in the Decision Space (IGDX) provide insights into convergence and solution diversity, respectively, but do not simultaneously capture performance across both spaces and frequently rely on subjective parameterization. This presents limitations in the comprehensive assessment of algorithm performance and adaptability to varying problem dimensions. To address these challenges, this paper proposes a new metric: the Dimension-Weighted Joint-Space Inverted Generational Distance (DWD). This metric integrates evaluations from both the decision and objective spaces. It exhibits robustness to variations in dimensionality and scale, operates without relying on adjustable parameters, and ultimately provides a more accurate reflection of an algorithm's multifaceted capabilities.**

***Index Terms*—Multimodal Multi-objective Evolutionary Algorithms, Performance Metrics, Joint-Space Evaluation, Dimensional Robustness**

I. Introduction

In recent years, the development and application of MMEAs have gained significant attention in the fields of optimization and artificial intelligence. MMEAs are designed to effectively navigate complex solution landscapes that feature multiple Pareto-optimal solutions, thereby addressing a variety of optimization problems with competing objectives. The ability of these algorithms to simultaneously find multiple optimal solutions holds great promise for real-world applications, ranging from engineering design and resource management to machine learning and data mining [1]–[3].

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An ideal solution set not only aims to approximate the true Pareto front (PF) in the objective space but also seeks to cover the true Pareto set (PS) in the decision space, all while maintaining a topological mapping between the two. Despite the advances in MMEAs, a critical challenge remains: the effective evaluation of these algorithms' performance. Current evaluation metrics often assess convergence and diversity in isolation, focusing solely on either the objective space or the decision space. For instance, the Inverted Generational Distance (IGD) [4] is a prevalent metric that quantifies convergence by measuring the distance between the obtained solution set and the true Pareto front, while the Inverted Generational Distance in the Decision Space (IGDX) [5] evaluates the diversity of solutions within the decision space. However, these metrics typically lack the capability to simultaneously capture algorithm performance across both domains, which limits their effectiveness in comprehensively assessing algorithm performance. More critically, when algorithms perform inconsistently across different metrics—for instance, achieving superior performance on IGD while exhibiting inferior results on IGDX—it becomes exceedingly difficult to adjudicate overall algorithmic superiority. This inconsistency in multi-metric evaluation creates ambiguity in determining which algorithm genuinely performs better, as existing metrics fail to provide a unified framework for reconciling conflicting performance indicators. Furthermore, many metrics rely heavily on subjective parameterization, introducing variability that may obscure true performance differences.

The limitations of existing metrics become even more pronounced when addressing the adaptability of algorithms to varying problem dimensions and complexities. As optimization problems grow increasingly intricate, the need for metrics that provide a holistic view of performance across both the objective and decision spaces becomes paramount. Traditional metrics often treat the decision space and the objective space as disjoint components, missing the opportunity to evaluate them in relation to one another as a unified whole. The disjoint evaluation approach not only prevents comprehensive performance assessment but also exacerbates the difficulty in making definitive comparisons when algorithms exhibit trade-offs between objective-space convergence and decision-space diversity. This challenge motivates the need for a new evaluation approach that transcends the constraints of conventional metrics.

To address these challenges, we propose the DWD, a novel metric designed to integrate evaluations of both decision and objective spaces. The core concept of this approach is to treat the decision and objective spaces as a cohesive entity, forming a Joint Space that captures the interplay and topological relationships between the two. By unifying both spaces into a single evaluative framework, DWD eliminates the inconsistency problem inherent in separate metrics and provides a definitive basis for comparing algorithm performance. The DWD is characterized by its robustness against variations in dimensionality and scale, as well as its independence from reference sets and adjustable parameters. By enhancing the interpretability and applicability of performance metrics, the DWD aims to provide a more accurate reflection of an algorithm's multifaceted capabilities, thereby facilitating better evaluations in complex decision-making scenarios and enabling clear, unambiguous adjudication of algorithmic superiority.

To validate the effectiveness of the DWD, we outline four experimental scenarios that will collectively demonstrate its advantages. The experiments will be conducted using the PlatEMO platform, which offers a comprehensive environment for benchmarking multimodal multi-objective optimization algorithms. Through these scenarios, we aim to illustrate the practical benefits of the proposed metric and its potential for advancing the evaluation of MMEAs.

II. Related Work

The evaluation of MMEAs is crucial for assessing their efficacy in solving problems that require finding multiple equivalent PSs in the decision space while approximating the PF in the objective space. Consequently, performance metrics must possess dual-space evaluation capabilities, measuring both convergence/diversity toward the PF and the coverage of multiple PSs.

Objective Space Metrics primarily assess proximity to the true PF and distribution uniformity. Widely adopted metrics include:

- **IGD** [4], which computes the average minimum distance from points on the true PF to the obtained solution set, serving as a convergence measure. The mathematical formulation of IGD is given by:

$$IGD(S) = \frac{1}{|S^*|} \sum_{y \in S^*} \min_{x \in X} \{ED(f(x), f(y))\} \quad (1)$$

  where ED(x, y) is the Euclidean distance between x and y. $\boldsymbol{S}$ and $\boldsymbol{S^*}$ represent the obtained solution set and a finite set of Pareto optimal solutions uniformly sampled from the true PF.
- **Hypervolume (HV)** [6], which quantifies the volume dominated by the solution set relative to a reference point, comprehensively reflecting convergence and diversity. However, HV is sensitive to reference point selection and suffers from exponential computational complexity in high dimensions.

$$HV(S, z^{\text{ref}}) = \Lambda \left(\bigcup_{x^* \in S^*} \{x \mid x^* \succ x \succ z^{\text{ref}}\} \right) \quad (2)$$

  where Λ denotes the Lebesgue measure and z^{ref} is the reference point in the objective space.

Decision Space Metrics evaluate an algorithm's ability to discover and preserve multiple equivalent PSs. Key metrics include:

- **IGDX** [5], an adaptation of IGD for the decision space, measuring the distance between the obtained PSs and the true PSs. The mathematical formulation of IGDX is given by:

$$IGDX(S) = \frac{1}{|S^*|} \sum_{y \in S^*} \min_{x \in S} \{ED(x, y)\} \quad (3)$$

  where ED(x, y) is the Euclidean distance between x and y. $\boldsymbol{S}$ and $\boldsymbol{S^*}$ represent the obtained solution set and a finite set of Pareto optimal solutions uniformly sampled from the true PS.
- **Coverage Rate (CR)** and **Pareto Sets Proximity (PSP)** [7], which gauge the similarity and coverage of the obtained PSs relative to the true ones. A derived metric, **RPSP (1/PSP)** [8], addresses the challenge that the optimal PSP value can approach infinity. The mathematical formulation of PSP and CR is given by:

$$CR(S) = \left(\prod_{i=1}^{D} \sigma_i \right)^{\frac{1}{2D}} \quad (4)$$

  where σ_i is the coverage of the PS for the ith dimension.

$$PSP(S) = \frac{CR(S)}{IGDX(S)} \quad (5)$$

Table I summarizes the metrics used in recent representative MMEA studies, highlighting the prevalent use of IGD and IGDX as core evaluators. In practice, researchers often employ a combination of metrics (e.g., IGD and IGDX) to evaluate performance in both spaces simultaneously. This situation reflects certain limitations in the current MMEA assessment framework, primarily the difficulty of relying on a single metric to fully evaluate algorithm performance.

Despite their utility, existing metrics have notable limitations. HV's high computational cost limits its use in many-objective problems. PSP's unbounded optimal value complicates performance interpretation. Composite metrics like

TABLE I: Performance metrics employed in the evaluation of multimodal multiobjective optimization algorithms.

Metrics	**TriMOEA-TA&R** [9]	**MMEA-WI** [10]	**HREA** [11]	**CoMMEA** [12]	**TSPXEA** [13]	**MMOWPA-CN** [14]	**LMMODE** [15]
IGD		Y	Y	Y		Y	Y
IGDX			Y	Y		Y	Y
HV					Y		Y
PSP	Y	Y					
IGDM	Y				Y		
GD					Y		
Metrics	**DN-NSGAII** [16]	**CPDEA** [17]	**DNEA-L** [18]	**MMEAPSL** [19]	**BOEA** [20]	**FPITSEA** [21]	**MMEA-TDI** [22]
IGD	Y				Y	Y	
IGDX	Y	Y		Y	Y	Y	
HV	Y		Y	Y	Y		Y
PSP				Y			Y
IGDM		Y					
GD							

IGDM [9] require user-defined weights to balance objective and decision space priorities, introducing subjectivity. Furthermore, when algorithms perform inconsistently across different metrics (e.g., Fig. 1a performs better on IGDX but worse on IGD, Fig. 1b performs better on IGD but worse on IGDX, and Fig. 1c performs well on both IGDX and IGD), it is difficult to judge overall superiority based on a single metric.

A fundamental challenge in the field is the lack of a robust, unified metric that intrinsically and meaningfully balances performance in both spaces. Current composite or combined metric approaches often fail to capture the nuanced trade-offs between objective-space quality and decision-space diversity. Therefore, the development of efficient, interpretable, and comprehensive performance indicators that seamlessly integrate dual-space evaluation remains a significant open issue and a vital direction for future research in Multimodal multiobjective optimization.

III. Dimension-Weighted Joint-Space Inverted Generational Distance

A. Design Philosophy and Core Concepts

In the field of Multimodal Multi-Objective Optimization (MMO), traditional evaluation metrics often face the dilemma of being overly simplistic or insufficiently informative. For instance, the classic IGD primarily focuses on the coverage of the objective space, which limits its ability to determine whether an algorithm effectively captures all relevant Pareto-optimal solutions. Even if an algorithm approaches the ideal PF, a lack of coverage across critical modalities renders IGD ineffective as a sole evaluative tool. Additionally, IGDX is highly sensitive to minimal variations in decision variables, and this sensitivity can lead to significant evaluation errors in high-dimensional contexts. Moreover, metrics such as CR and PSP typically require external parameters or thresholds, undermining their claim to parametric independence.

To address these challenges, we propose a novel evaluation metric. This metric aims to effectively amalgamate assessments from both the objective and decision spaces, resulting in a comprehensive evaluation criterion that operates without reliance on parameters. By utilizing statistical properties, DWD mitigates the influence of scale and dimensionality, thereby achieving a thorough evaluation of the solution set.

B. Metric Definition

The design of DWD encompasses several key steps, the first step involves dynamically normalizing the solution set based on the statistical characteristics (such as maximum and minimum values) of the true reference set. This process ensures uniformity across the overall evaluation standard and is represented by the following equations:

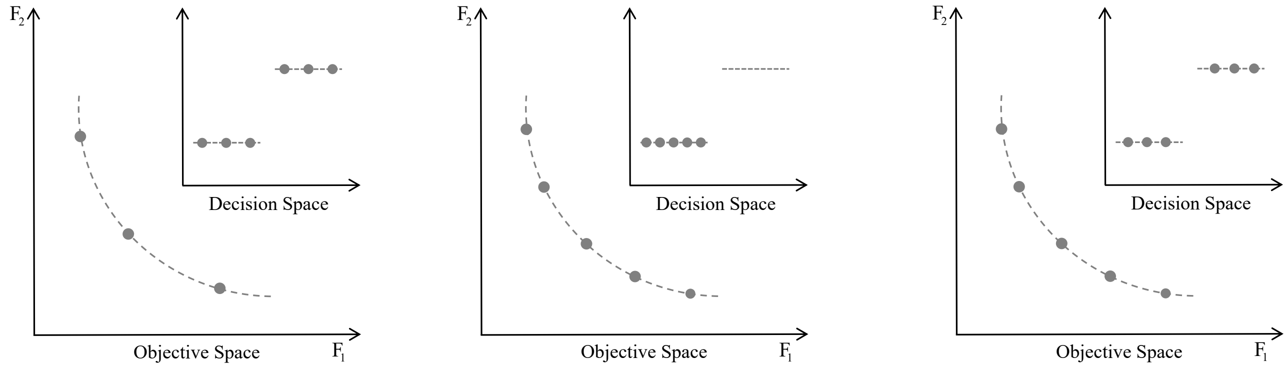
$$x_i = \frac{x_i - \min(P_x^*)}{\max(P_x^*) - \min(P_x^*)} \quad i = 1, \ldots, n \tag{6}$$

$$y_j = \frac{y_j - \min(P_y^*)}{\max(P_y^*) - \min(P_y^*)} \quad j = 1, \ldots, m \tag{7}$$

Let $\boldsymbol{P^*}$ be the reference set containing N solutions, each solution consists of a decision vector $\mathbf{x}^*$ and an objective vector $\mathbf{y}^*$, where n is the dimension of the decision variables and m is the dimension of the objective functions.

To prevent the high-dimensional decision space from overwhelming the low-dimensional objective space, the DWD method addresses this by incorporating weighting factors based on the inverse of the respective dimensions. The joint distance is thus defined as:

$$D_{\text{joint}}(r, s) = \sqrt{\frac{1}{n}\sum_{k=1}^{n}(x_{r,k} - x_{s,k})^2 + \frac{1}{m}\sum_{k=1}^{m}(y_{r,k} - y_{s,k})^2} \tag{8}$$



(a) Case 1 (b) Case 2 (c) Case 3

Fig. 1: The schematic of population distribution in case 1 exhibits good distribution in the decision space but poor distribution in the objective space, while case 2 displays good distribution in the objective space but poor distribution in the decision space, and case 3 shows good distribution in both the objective and decision spaces.

where $r \in \boldsymbol{P^*}$ is a reference solution and $s \in \boldsymbol{S}$ is a solution from the solution set S generated by the algorithm.

$\frac{1}{n}$ and $\frac{1}{m}$ ensure that the contributions of the decision space and objective space to the total distance are "equal opportunities." Regardless the number of dimensions, they collectively (decision quality) hold equal importance as the quality of the objectives.

The calculation process for DWD resembles that of IGD, where each reference point is iteratively examined to identify the nearest point within the algorithm's solution set. The resulting aggregated metric is expressed as:

$$DWD(S, P^*) = \frac{1}{|P^*|} \sum_{r \in P^*} \min_{s \in S} D_{\text{joint}}(r, s) \tag{9}$$

IV. Experiments

A. Experimental Design

In this study, we evaluate the performance of the proposed DWD metric within the context of MMO. Our experiments are designed to address the challenges inherent in evaluating both convergence and diversity in solution sets.

To conduct a comprehensive assessment, we categorize the experiments into four distinct scenarios, each representing different characteristics of test problems and algorithms. This structured approach allows us to analyze the strengths and weaknesses of various algorithms in terms of their ability to achieve high-quality solutions across multiple objectives and decision dimensions.

Scenario A: For scenario where the algorithm exhibits excellent convergence but struggles to obtain multiple optimal solutions, the IDMP test suite [23] is used to demonstrate its limitations. In this problem, equivalent Pareto optimal solutions possess varying degrees of computational difficulty.

Scenario B: For scenario where the algorithm successfully identifies all modal regions but exhibits poor convergence, the IDMP_e test suite [11] highlights its shortcomings. This problem set contains locally structured Pareto frontiers, where searching within such regions degrades the algorithm's global convergence performance.

Scenario C: This scenario involves test problems characterized by high-dimensional decision variables. We use the HDMMF test suite [24] to illustrate this limitation, as solving these test problems entails navigating a high-dimensional decision space.

Scenario D: This final scenario involves perfect solution sets represented by CEC 2019 test suite [7] compared against various algorithms.

In these scenarios, our aim is to comprehensively evaluate the advantages of the DWD metric in assessing solution quality under different circumstances. To achieve this, we selected nine algorithms—CoMMEA [12], DN-NSGAII [16], HREA [11], MMEA-WI [10], MO_Ring_PSO_SCD [7] (abbreviated as MRPS in tables for clarity), Omni-optimizer [25] (abbreviated as Omni in tables for clarity), DNEA [26], DNEAL [18], and HDMMODE [24]—to solve the test problems, in order to systematically analyze the impact of population distribution on the performance of the metrics in different scenarios.

For each scenario, we will center around a representative test case to visualize the population distribution generated by the representative algorithm. Additionally, we will present a summary comparison table of the algorithm's performance metrics in each scenario. The analysis will include key metrics such as IGD, IGDX, and DWD to draw conclusions about the effectiveness of these metrics.

The experiments will be conducted on the PlatEMO platform [27]. Each algorithm will be independently executed for 30 runs to ensure the reliability and robustness of the results. The running environment will utilize an Intel(R) Core(TM) i7-14700F processor.

B. Results and Analysis

The results will highlight the effectiveness of the DWD metric compared to traditional metrics like IGD and IGDX in MMO settings. By measuring various aspects of the solution distribution, we anticipate verifying that our proposed metric effectively addresses the limitations of existing measures, especially in scenarios characterized by high dimensionality and diverse solution landscapes.

1) Scenario A: As shown in Figures 2 and 3, there is a small difference between the DWD and IGDX metrics, as well as between the IGDM and IGD metrics. Notably, in Scenario A, the lack of diversity in the decision space is a key factor affecting algorithm performance. Therefore, metrics focused on decision space performance more accurately reflect algorithm effectiveness. When decision space diversity is limited, the evaluations of DWD and IGDX converge, further demonstrating the validity of the DWD metric.

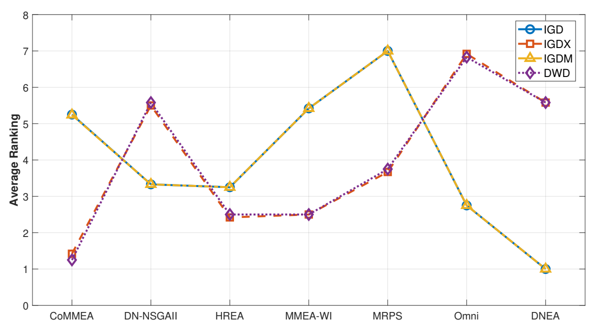


Fig. 2: Metric rankings of different algorithms on IDMP test problems.

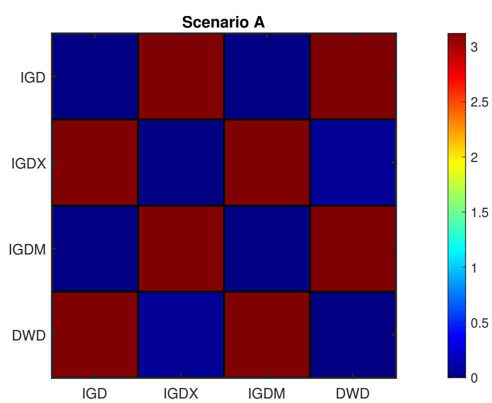


Fig. 3: Heat map of metric Differences on IDMP test problems.

Fig. 4 presents the population distributions obtained by DNEA and CoMMEA in the IDMPM2T2 test problem. As seen in the figure, both algorithms successfully converged to the PF; however, DNEA was only able to identify a partial PS. This observation is also reflected in the relevant evaluation metrics. Although one might be inclined to select DNEA based on the existing metric system, it is evident that CoMMEA exhibits superior performance in terms of diversity and convergence. This conclusion is consistent with the results obtained through our constructed joint space indicators.

2) Scenario B: As illustrated in Fig. 5 and Fig. 6, the differences among the DWD, IGDX, and IGDM metrics are marginal. Moreover, although the ranking of the IGD metric is inconsistent with those of the other three metrics, the

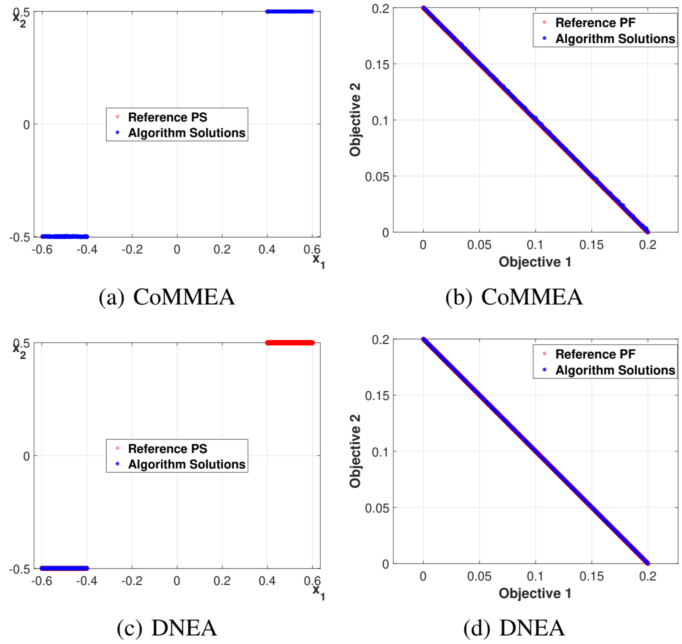


Fig. 4: Population distribution of algorithms in IDMPM2T2 test problem

actual IGD values of the compared algorithms are very close. These observations suggest that existing multimodal multi-objective test problems featuring local Pareto fronts may be insufficient for adequately evaluating algorithmic performance, particularly in terms of validating convergence in the objective space.

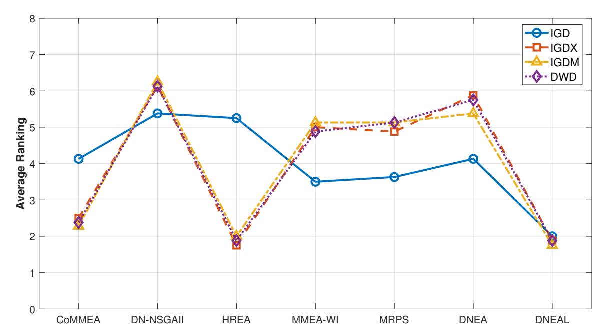


Fig. 5: Metric rankings of different algorithms on IDMP_e test problems.

Fig. 7 illustrates the population distributions obtained by HREA and DNEAL on the IDMPM2T2_e test problem. The results show that while both algorithms successfully identify the complete Pareto set (PS), DNEAL retains only a portion of the Pareto front (PF). In contrast, the evaluation metrics reveal a different trend—HREA ranks lower than DNEAL in terms of the IGD metric, with their metric values being relatively close—which poses challenges for decision-makers in selecting the most suitable algorithm. Notably, in Scenario B, the objective is to identify all modal regions within the problem; therefore, performance metrics focused on the objective space

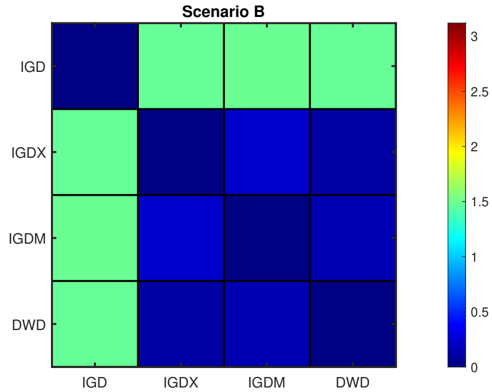
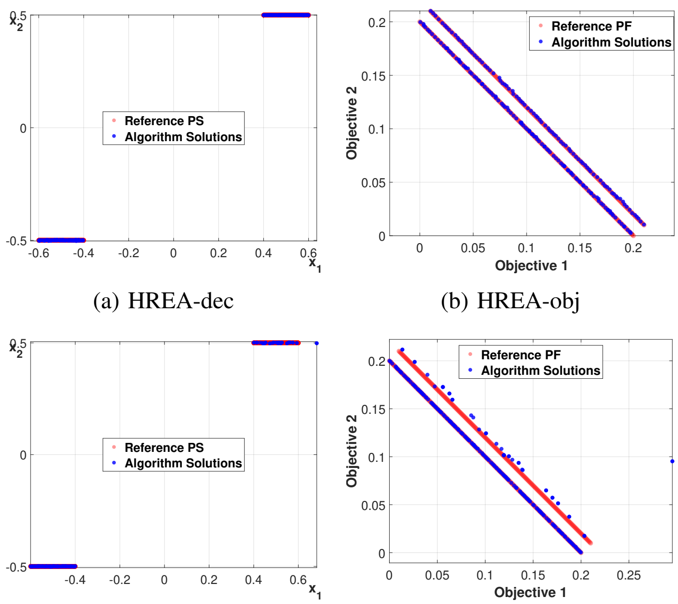


Fig. 6: Heat map of metric Differences on IDMP_e test problems.



(a) HREA-dec (b) HREA-obj

(c) DNEAL-dec (d) DNEAL-obj

Fig. 7: Population distribution of algorithms in IDMPM2T2_e test problem

are more effective at accurately reflecting algorithm performance. This observation further highlights the limitations of existing multimodal multi-objective test problems with local Pareto fronts, as they fail to adequately address the increasing sophistication of algorithmic performance.

3) Scenario C: As shown in Fig. 8 and Fig. 9, HDMMODE demonstrates a significant advantage when evaluated using our proposed metrics. This suggests that while existing metrics highlight specific strengths of certain algorithms, our new metrics reveal a more robust superiority for HDMMODE, providing a clearer indication of its effectiveness in the current high-dimensional optimization setting.

Fig. 10 illustrates the population distribution obtained by HDMMODE and Omni in the HDMMF2 test problem. It can be observed that HDMMODE demonstrates superior performance in maintaining the PS and the PF, a trend that is

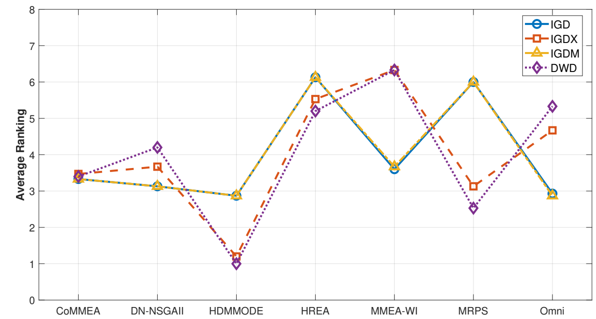


Fig. 8: Metric rankings of different algorithms on HDMMF test problems.

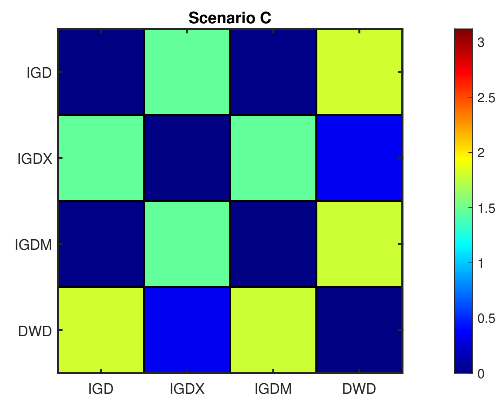
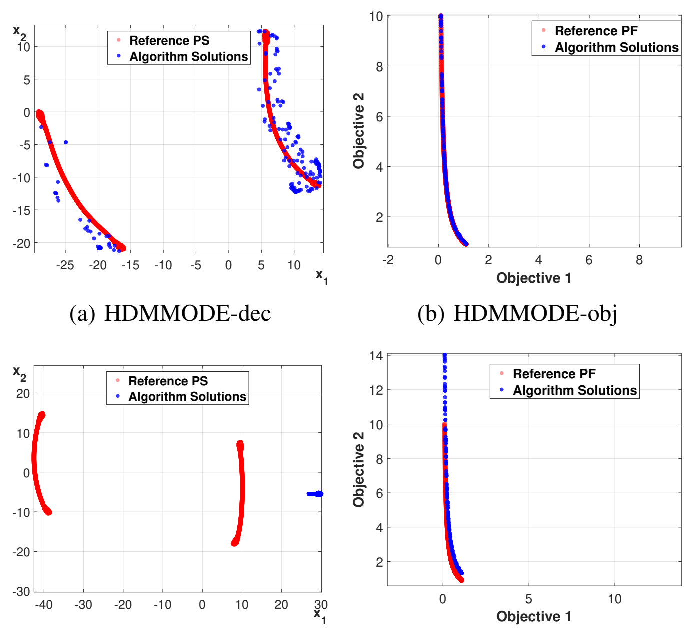


Fig. 9: Heat map of metric Differences on HDMMF test problems.



(a) HDMMODE-dec (b) HDMMODE-obj

(c) Omni-dec (d) Omni-obj

Fig. 10: Population distribution of algorithms in the HDMMF2 test problem, visualized using t-SNE dimensionality reduction in the decision space

reflected in various evaluation metrics. Notably, while the population distribution achieved by the Omni algorithm in the decision space deviates significantly from the true PS, its metric rankings remain comparatively high. We believe this discrepancy may be attributed to the influence of the curse of dimensionality on the accuracy of the IGDX metric calculations in high-dimensional decision spaces. In contrast, the dimension-balancing strategy employed by DWD more effectively assesses the true performance of the population in the joint space.

4) Scenario D: As shown in Fig. 11 and Fig. 12, the evaluation results of the DWD metric differ from those of the other three metrics. This discrepancy arises from the characteristics of Scenario D, a relatively simple multimodal multi-objective optimization problem where algorithms can easily identify all modes and achieve good convergence, leading to strong overall performance across metrics. However, this advantage can also cause conflicts in the emphasis of different metrics during evaluation.

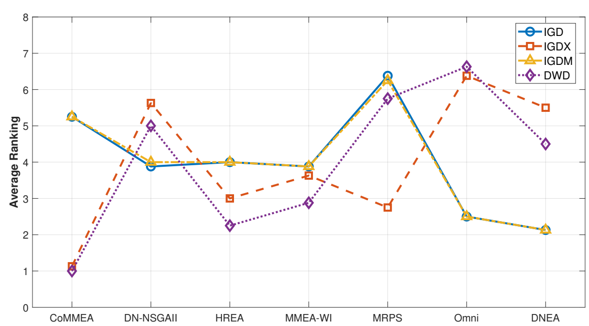


Fig. 11: Metric rankings of different algorithms on CEC 2019 test problems.

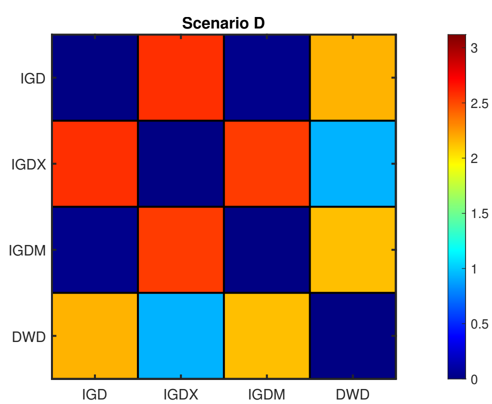
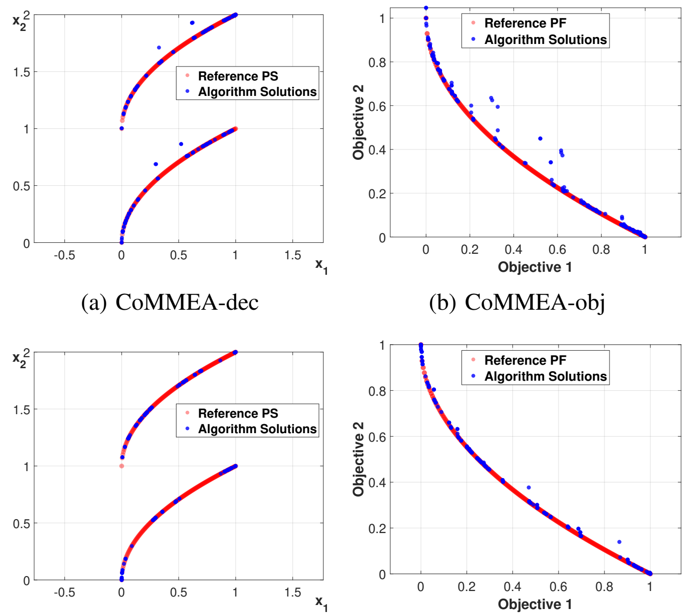


Fig. 12: Heat map of metric Differences on CEC 2019 test problems.

Fig. 13 illustrates the population distribution obtained by CoMMEA and DNEA in the MMF2 test problem. It is evident that both algorithms perform well in maintaining the PS and the PF, and this advantage is also reflected in various evaluation metrics. However, as the population distributions achieved by both methods are quite similar, it is challenging to directly determine which algorithm is superior. In this context, the assessment provided by DWD in the joint space serves as an effective criterion; this metric can clearly distinguish the performance of the algorithms across different test problems, thereby helping us identify the algorithm that exhibits better overall performance in the joint space.



(a) CoMMEA-dec (b) CoMMEA-obj

(c) DNEA-dec (d) DNEA-obj

Fig. 13: Population distribution of algorithms in MMF2 test problem

V. Conclusions

The DWD metric embodies the design philosophy of "balance and unity," effectively addressing the challenges in MMO evaluation.

The metric is parameter-free, relying solely on the statistical boundaries of the reference set. This eliminates the ambiguity associated with parameter selection. By utilizing a joint-space approach, the DWD ensures that weaknesses in either decision variables or objective functions negatively affect the overall score, thus providing a holistic assessment of algorithm performance. It employs dimension weighting to resolve issues where high-dimensional decision variables can obscure convergence of objective functions, maintaining sensitivity across dimensions.

Overall, the DWD is particularly suited for evaluating algorithms that need to achieve high-precision convergence while preserving diversity in decision spaces, making it an effective tool for complex multi-modal optimization tasks.

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