GEvTO: A Hybrid Evolutionary Scheme for Large-Scale Multi-objective Binary Topology Optimization of Electromagnetic Devices

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***Abstract*—This paper presents a hybrid multi-objective framework for large-scale topology optimization of electromagnetic devices characterized by binary decision variables. The proposed method, termed Gradient Evolutionary Topology Optimization-strategy with a local gradient-based search formulated as an integer linear programming problem. The evolutionary component is responsible for global exploration of the design space through an adaptive mutation operator, while a filtering strategy mitigates checkerboard patterns and improves topology connectivity. The local gradient search, activated at prescribed generations, refines the candidate solution. The effectiveness of the proposed approach is demonstrated through the constrained multi-objective topology optimization of the receiver ferrite plate in a wireless power transfer system. Numerical results demonstrate that GEvTO can generate a well-resolved Pareto front and produce manufacturable topologies.**

***Index Terms*—Topology optimization, multi-objective optimization, large scale optimization, Pareto-optimal solutions, evolutionary computation, linear programming, electromagnetics**

I. Introduction

Pareto-based solutions exploiting stochastic algorithms have been proposed in recent years in the context of topology optimization (TO) problems, however, in this field, the weighted-sum (WS) method remains the state of the art for solving multi-objective problems (MOPs) across a wide range of physical domains, even though it is considered outdated in the context of parametric optimization due to its well-known limitations [1].

As shown in Fig. 1, in TO, the n decision variables $\mathbf{x} \in \Omega_{\mathbf{x}}$ map the presence or absence of a given material within the design domain Ω_d. The goal of TO is to determine a material distribution that satisfies the MOP, often by imposing a constraint on the available amount of material. To do so, Ω_d is discretized into n discrete elements (mesh). As a consequence, TO problems are inherently large-scale, since realistic engineering applications typically require thousands of mesh elements to ensure high accuracy.

The evaluation of the objective functions $\mathbf{f} = [f_1, \ldots, f_m]^\top \in \Omega_{\mathbf{f}} \subset \mathbb{R}^m$ requires the solution of the forward electromagnetic (EM) problem $\mathcal{F} : \Omega_{\mathbf{x}} \to \Omega_{\mathbf{f}}$.

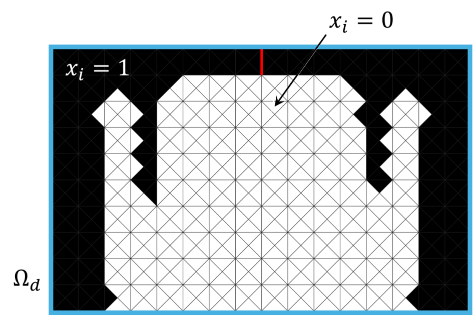

Fig. 1. Example of TO. $\mathbf{x} = [0, 0, 1, 1, 0, 1, \ldots, 0]^\top$, meaning that the material is present in the filled regions ($x_i = 1$), while it is absent in the void parts ($x_i = 0$) of Ω_d.

Standard evolutionary algorithms (EA) are usually inconvenient due to the high computational cost of computing $\mathbf{f} = \mathcal{F}(\mathbf{x})$. Moreover, without proper guidance, individuals may exhibit poorly shaped, checkerboard-like topologies that are unrealistic for manufacturing.

On the contrary, gradient-based algorithms, even if they do not guarantee convergence to the global optimum, are usually preferred for performing TO [2]. However, these approaches require sensitivity evaluations (i.e., $\mathrm{d}f_i/\mathrm{d}\mathbf{x}\ \forall i = 1, \ldots, m$), thereby increasing the computational cost.

Hybrid strategies combining EAs and gradient-based methods represent an attractive solution to solve MOPs in the context of TO. Hybrid single-objective algorithms combining evolutionary and gradient-based search have been proposed in the literature [3], [4]. Here, the gradient method is used as a local search to refine the topologies generated by the evolutionary scheme and smooth their boundaries.

In this paper, a hybrid multi-objective topology optimization approach dealing with binary decision variables, termed GEvTO (Gradient Evolutionary Topology Optimization), is presented. The key contributions of the GEvTO are summarized here:

1) *Evolutionary scheme*: A differential evolution (DE) scheme involving adaptive mutation and crossover/recombination is applied to explore the search

The raw data supporting the conclusions of this article will be made available by the authors on request.

space. A filtering strategy is applied to the trial vector to smooth out the corresponding topology to improve the element connectivity.
2) *Local gradient search*: A tunable local gradient search is used to fix the topologies generated by the evolutionary scheme.
3) *Integer linear programming*: is used to advance the decision variables with $\Omega_{\mathbf{x}} = \{0,1\}^n$ during the gradient search.
4) *Pareto-optimal solutions*: The algorithm exploits the nondominated sorting and crowding distance strategies to construct Pareto-optimal solutions. A constraint domination principle is used to find feasible solutions.

The remainder of the paper is organized as follows. Section II describes the EM model associated with the forward problem, with particular emphasis on model selection and objective evaluation. The multi-objective problem is fully presented in Section III, where the key components of the GEvTO algorithm are detailed.

The proposed algorithm is then applied to the TO of the ferrite plate in a Wireless Power Transfer (WPT) device. Although ferrite is the predominant ferromagnetic material used in WPT coil assemblies, it has notable drawbacks, including brittleness, high expense, and substantial weight. Therefore, decreasing reliance on ferrite without compromising performance represents a significant research challenge, for which TO offers a promising design approach. The resulting Pareto front is analyzed in Section IV. Conclusions and future research directions are discussed in Section V.

II. Electromagnetic Analysis

A. Issue of Model Selection

Generally, the numerical solution of $\mathcal{F}(\mathbf{x})$ is not straightforward due to the variety of EM problems. Moreover, solution approaches are not unique, e.g., the FEM is advantageous for problems where the EM field lines are confined to a specific region (e.g., electric motors). Conversely, the Integral Equation Method (IEM) is attractive for open-boundary problems (e.g., antennas). Both approaches have been used in the context of TO [5], [6].

In the linear time-independent case, $\mathcal{F}(\mathbf{x})$, in the discrete setting, can be written as:

$$\mathbf{S}(\mathbf{x})\mathbf{y} = \mathbf{b}, \quad \mathbf{f} = \mathcal{M}(\mathbf{y}) \tag{1}$$

where $\mathbf{S}$ is the system matrix, counterpart of the Partial Differential Equation (PDE) governing the EM problem in $\Omega \subset \mathbb{R}^3$, $\mathbf{y}$ is the array of real or complex unknowns, and $\mathbf{b}$ is the vector of known terms.

The array of objectives, $\mathbf{f}$, is obtained through an additional mapping from the solution vector. In what follows, the explicit dependence on $\mathbf{x}$ is omitted; thus, objectives of the form $f_i(\mathbf{y}(\mathbf{x}))$ are considered. To simplify the notation, we will simply write $f_i(\mathbf{x})$ in the remainder of the article. Notably, $\mathcal{M}(\mathbf{y})$ can be different if FEM or IEM is used, even if the same physical problem is solved, due to the different meaning of $\mathbf{y}$.

In this research, the EM problem is solved assuming magneto-quasistatic conditions in the frequency domain, at operational frequency f_{op} [Hz]. Numerous formulations have been proposed in the literature for solving such problems [7], each with its own pros and cons, which affect the efficacy of the forward problem solution, and, consequently, the speed and accuracy of the entire optimization task. Here, the A-formulation, expressed in terms of the complex magnetic vector potential $\mathbf{A}(\mathbf{r})$, is adopted [7]:

$$\nabla \times \left(\mu^{-1}\nabla \times \mathbf{A}(\mathbf{r})\right) + i2\pi f_{op}\rho^{-1}\mathbf{A}(\mathbf{r}) = \mathbf{J}_{ext}, \tag{2}$$

where i is the imaginary unit, $\mathbf{J}_{ext}$ is the imposed current density, and ρ is the electric conductivity. Exploiting the Galerkin testing procedure the system of equations of Eq. (1) is assembled. In the subsequent analysis, linear magnetic materials with $\mu = \mu_0\mu_r$ are considered, where μ is the magnetic permeability, μ_0 is the vacuum permeability, and μ_r is the relative permeability.

III. Multi-objective Optimization

A. Preliminary on Topology Optimization

In this research, the TO aims to find the distribution of magnetic material in Ω_d. In this case, μ_r for the ith element discretizing Ω_d, is obtained from x_i through the map:

$$\mu_{r,i} = \mu_{r,min} + (\mu_{r,max} - \mu_{r,min})x_i, \tag{3}$$

where $\mu_{r,min} = 1$ and $\mu_{r,max} = 3500$. This means that $x_i = 0$ corresponds to void (i.e., air) elements, while $x_i = 1$ corresponds to elements filled with magnetic material (ferrite). Using (3), the map $\mathbf{x} \mapsto \mathbf{S}(\mathbf{x})$ can be defined.

A popular constraint added to TO is used to limit the amount of material filling Ω_d, written as:

$$G(\mathbf{x}) = \frac{V(\mathbf{x})}{V_{\Omega_d}} = \frac{\sum_{i=1}^{n} x_i vol_i}{V_{\Omega_d}} \leq \bar{G} \tag{4}$$

where vol_i is the volume of ith mesh element, while V_{Ω_d} is the total volume of Ω_d. As an example, for electric vehicles, reducing the amount of ferrite leads to a reduction in the weight and costs of the WPT system onboard.

B. Differential Evolution Strategy

We consider a DE with a population $\mathcal{P}$ of N individuals $\mathbf{X}_i = [X_{1,i}, X_{2,i}, \ldots, X_{n,i}]$ $(i = 1, \ldots, N)$, where $X_{j,i} \in [-1, 1]$.

1) Mutation operator: Starting from the ith individual $\mathbf{X}_{i,g}$ of the parent population at the generation g, the mutation operator DE/current-to-$p$best/1 proposed in the JADE scheme is used to build the donor vector $\mathbf{v}_{i,g}$ [8]:

$$\mathbf{v}_{i,g} = \mathbf{X}_{i,g} + F_i(\mathbf{X}^p_{best,g} - \mathbf{X}_{i,g}) + F_i(\mathbf{X}_{r1,g} - \mathbf{X}_{r2,g}), \tag{5}$$

adaptively controlling F_i by exploiting a Cauchy distribution:

$$F_i = \mathrm{randc}_i(\mu_F, \sigma_F), \tag{6}$$

where σ_F is the scale parameter and $\mu_F = (1-c)\mu_F + \mathrm{mean}_L(S_F)c$, with $\mathrm{mean}_L(\cdot)$ representing the Lehmer mean [8]. In the multi-objective setting, S_F is an N-dimensional

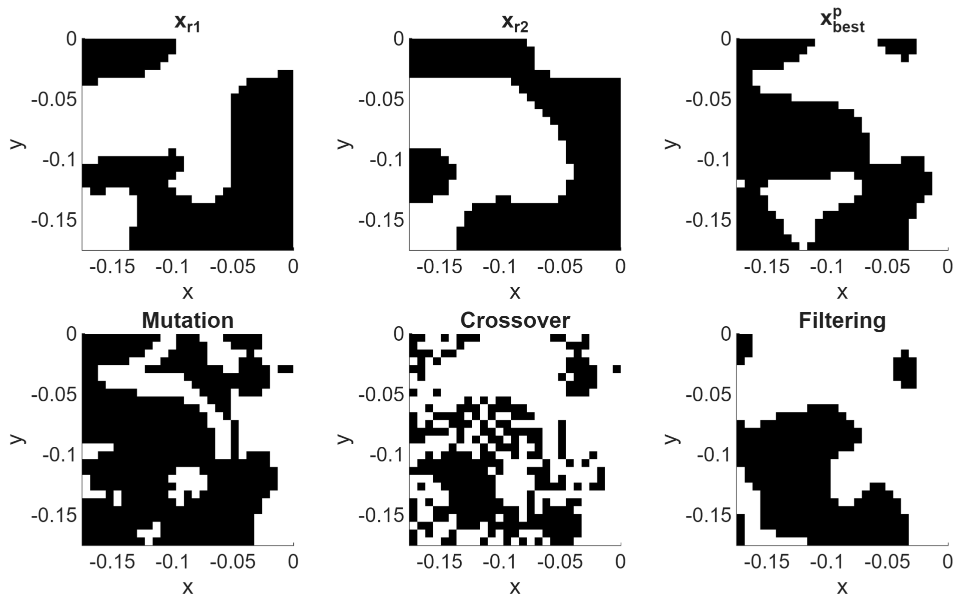

Fig. 2. Example of mutation, crossover, and filtering starting from the individuals $\mathbf{X}_{r1}, \mathbf{X}_{r2}$ and $\mathbf{X}^{p}_{best}$.

array of the indices of the first N best individuals in the archive ($\mathcal{A}$). In (5), $r1 \neq r2$ are random integers in $\{1, \ldots, N\}$ while $\mathbf{X}^{p}_{best,g}$ is randomly chosen from the top $100p\%$ individuals in the population. A graphical example of the donor vector can be seen in Fig. 2.

2) Crossover/Recombination: A binomial crossover operator controlled by the constant $CR \in [0, 1]$ is used to build the trial/offspring vector $\mathbf{u}_{i,g} = [u_{1,i,g}, u_{2,i,g}, \ldots, u_{n,i,g}]$:

$$u_{j,i,g} = \begin{cases} v_{j,i,g} & \text{if rand(0,1)} \leq CR \text{ or } j = j_{rand} \\ X_{j,i,g} & \text{otherwise,} \end{cases} \tag{7}$$

where rand(0,1)$\sim \mathcal{U}(0, 1)$, and j_{rand} is a random index within $\{1, \ldots, n\}$ newly generated for each i [9]. A repair rule is used to ensure $u_{j,i}$ within the bounds, then a sigmoid function:

$$\sigma(u_{j,i}) = \frac{2}{1 + \exp\left(-2u_{j,i}\right)} - 1, \tag{8}$$

is used to recast the real variables into the binary space. Let $\mathbf{I}^{-}$ denote the index set of variables for which $\sigma(\mathbf{u}(\mathbf{I}^{-})) < 0$; conversely, let $\mathbf{I}^{+}$ denote the index set of variables for which $\sigma(\mathbf{u}(\mathbf{I}^{+})) > 0$. Then, $\mathbf{u}(\mathbf{I}^{-}) = 0$ and $\mathbf{u}(\mathbf{I}^{+}) = 1$. An example of the trial vector is shown in Fig. 2.

3) Filtering/Blurring: It is not granted that $\mathbf{u}_i$ has good connectivity properties. Checkerboard patterns may compromise the structural integrity of the optimized component and may result in an unmanufacturable design. A manufacturability constraint can be enforced by solving an additional physical problem (e.g., structural mechanics), further increasing the computational cost unless surrogate models are used [10]. Also, stochastic algorithms optimizing a normalized Gaussian network (NGnet) can be applied [11].

Structural connectivity is not a mandatory requirement. While it plays a central role in the TO of electric motors [12], novel manufacturing approaches in patch antenna design enable the optimized design to be printed directly onto the substrate [13]. Following [14], a blurring/filtering strategy is adopted to reduce the checkerboard pattern while keeping the same number of elements with $u_{j,i} = 1$. Details of this approach can be found in [14]. The outcome of this procedure can be seen in Fig. 2. Note that the corresponding topology exhibits good connectivity, with a few isolated islands.

The decision variables output of this procedure are then used for the TO mapping (3).

C. Local Gradient Search

The evolutionary scheme is hybridized with a local gradient-based search [15], [16] to further refine the topology after the filtering stage. The algorithm exploits the Sequential Approximate Integer Programming (SAIP) [17], briefly recalled considering a single-objective optimization with objective function $f(\mathbf{x}) = f_i(\mathbf{x})$ $(i = 1, \ldots, m)$, under the inequality constraint $G(\mathbf{x}) \leq \bar{G}$. In SAIP, a truncated Taylor expansion of the objective and constraints is used to approximate the functions in the neighborhood of the decision vector at kth iteration ($\mathbf{x}^k$):

$$f(\mathbf{x}) \approx f(\mathbf{x}^k) + \left.\frac{\mathrm{d}f}{\mathrm{d}\mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^k} \cdot \Delta\mathbf{x}, \tag{9}$$

where $\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}^k$. The original single-objective optimization is transformed into the following sequence of Integer Linear Programming (ILP) problems:

$$\begin{aligned} \min_{\Delta\mathbf{x}} \quad & \left.\frac{\mathrm{d}f}{\mathrm{d}\mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^k} \cdot \Delta\mathbf{x} \\ \text{s.t.} \quad & \left.\frac{\mathrm{d}G}{\mathrm{d}\mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^k} \cdot \Delta\mathbf{x} \leq \bar{G} - G(\mathbf{x}^k) \\ & \|\Delta\mathbf{x}\|_1 \leq n\beta_{flip}, \end{aligned} \tag{10}$$

Algorithm 1: Framework of GEvTO algorithm

Input: n, N, $\bar{G}$, g_{grad};
Output: $\mathcal{A}$;
$g \leftarrow 0$; /* generation counter */
$\mathcal{F}_1 \leftarrow \emptyset$;
$\mathcal{F}_2 \leftarrow \emptyset$;
$\mathcal{G} \leftarrow \emptyset$;
$\mathcal{P}^0 \leftarrow Initialize(n, N)$; /* random initialization */
for $\mathbf{x}_{i,0} \in \mathcal{P}^0$ **do**
  evaluate objectives and constraints;
  $\mathcal{F}_1 \leftarrow \mathcal{F}_1 \cup f_1(\mathbf{x}_{i,0})$;
  $\mathcal{F}_2 \leftarrow \mathcal{F}_2 \cup f_2(\mathbf{x}_{i,0})$;
  $\mathcal{G} \leftarrow \mathcal{G} \cup G(\mathbf{x}_{i,0})$;
end
$\mathcal{A} \leftarrow \emptyset$; /* initialize archive */
put $\mathcal{P}^0$, $\mathcal{F}_1$, $\mathcal{F}_2$, and $\mathcal{G}$ into $\mathcal{A}$;
while *termination criterion is not fulfilled* **do**
  extract N individuals from $\mathcal{A}$;
  $\mathcal{F}_1 \leftarrow \emptyset$;
  $\mathcal{F}_2 \leftarrow \emptyset$;
  $\mathcal{G} \leftarrow \emptyset$;
  for *each individual* **do**
    perform mutation according to (5);
    perform recombination according to (7);
    perform filtering of trial vector;
    evaluate objectives and constraints;
    if $rem(g, g_{grad}) = 0$ **then**
      for *each trial vector* **do**
        compute new individual $\tilde{\mathbf{u}}_{f1,i,g}$ with ILP following $\mathrm{d}f_1$ with constraint in G and $f2$;
        $\mathcal{F}_1 \leftarrow \mathcal{F}_1 \cup f_1(\tilde{\mathbf{u}}_{f1,i,g})$;
        $\mathcal{F}_2 \leftarrow \mathcal{F}_2 \cup f_2(\tilde{\mathbf{u}}_{f1,i,g})$;
        $\mathcal{G} \leftarrow \mathcal{G} \cup G(\tilde{\mathbf{u}}_{f1,i,g})$;
        compute new individual $\tilde{\mathbf{u}}_{f2,i,g}$ with ILP following $\mathrm{d}f_2$ with constraint in G and $f1$;
        $\mathcal{F}_1 \leftarrow \mathcal{F}_1 \cup f_1(\tilde{\mathbf{u}}_{f2,i,g})$;
        $\mathcal{F}_2 \leftarrow \mathcal{F}_2 \cup f_2(\tilde{\mathbf{u}}_{f2,i,g})$;
        $\mathcal{G} \leftarrow \mathcal{G} \cup G(\tilde{\mathbf{u}}_{f2,i,g})$;
      end
    end
    $\mathcal{P}^g \leftarrow$ new individuals;
  end
  put $\mathcal{P}^g$, $\mathcal{F}_1$, $\mathcal{F}_2$, $\mathcal{G}$ into the archive $\mathcal{A}$;
  $g \leftarrow g + 1$;
end
return output

where β_{flip} controls the number of decision variables flipping from one iteration to another. Solving (10), $\mathbf{x}^{k+1} = \mathbf{x}^k + \Delta\mathbf{x}$.

The Adjoint Variable Method (AVM) is used to compute $\mathrm{d}f/\mathrm{d}\mathbf{x}$ [18]. During the forward problem, the AVM requires the solution of an additional system of equations for the adjoint variable $\boldsymbol{\lambda}$ expressed as: $\mathbf{S}^\top\boldsymbol{\lambda} = -\partial f/\partial\mathbf{y}$, from which the sensitivity is computed:

$$\frac{\mathrm{d}f}{\mathrm{d}x_i} = \frac{\partial f}{\partial x_i} + \boldsymbol{\lambda}^\top \frac{\partial \mathbf{S}(\mathbf{x})}{\partial x_i}\mathbf{y}. \tag{11}$$

It is worth noting that for the constraint in (4), the evaluation of $\mathrm{d}G/\mathrm{d}\mathbf{x}$ is trivial, being:

$$\frac{\mathrm{d}G(\mathbf{x})}{\mathrm{d}\mathbf{x}} = \frac{[vol_1, vol_2, \ldots, vol_n]^\top}{V_{\Omega_d}}. \tag{12}$$

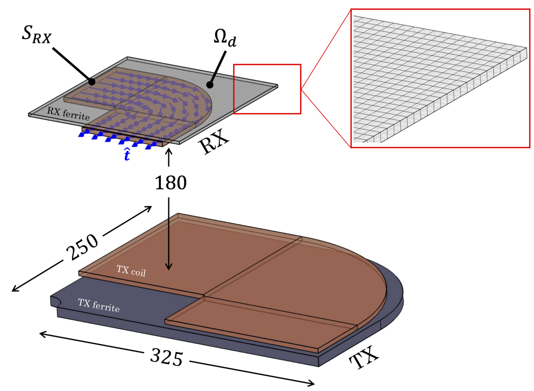

Fig. 3. Geometry of WPT device under analysis. For symmetry reasons, only 1/4 of the domain is shown. A detail of the hexahedral finite element mesh used for Ω_d is also shown. Units in mm.

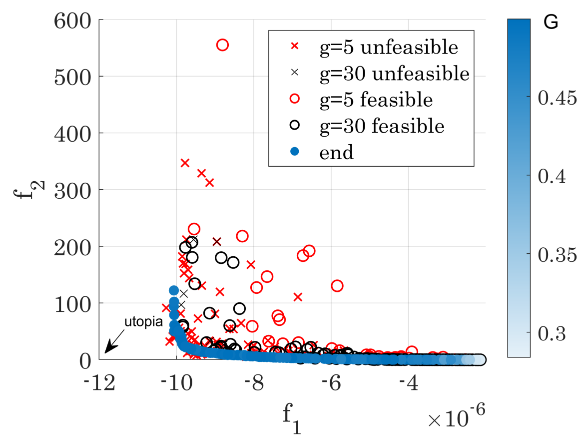

Fig. 4. Objective space during iterations of the GEvTO algorithm. The colorbar refers to the value of $G(\mathbf{x})$ of each solution at the end of GEvTO iterations.

D. The GEvTO Algorithm

The pseudo code of the GEvTO algorithm is listed in Alg. 1. A random population $\mathcal{P}^0 = [\mathbf{x}_1^0, \mathbf{x}_2^0, \ldots, \mathbf{x}_N^0]$ of N individuals is generated at the beginning. The objectives and constraints are evaluated and their values stored in the archive $\mathcal{A}$ together with $\mathcal{P}^0$. According to the NSGA-II [19], $\mathcal{A}$ is organized according to the non-dominated sorting and crowding distance strategies.

At each iteration g, the individuals in the current population $\mathcal{P}^g$ extracted from $\mathcal{A}$, are subjected to the mutation, crossover, and filtering operations explained in Section III-B. The objective functions and constraints are then evaluated for each filtered trial vector. Every g_{grad} iterations, the forward solver also provides the values of $\mathrm{d}f_i/\mathrm{d}\mathbf{x}$ to be used in the local gradient search. Within the local gradient search, the ILP (10) is used to find a new individual starting from the filtered trial vector. First, the new individual $\tilde{\mathbf{u}}_{f1,i,g}$ is computed by moving along the direction dictated by $\mathrm{d}f_1/\mathrm{d}\mathbf{u}_{i,g}$. An additional constraint is added to (10) to control the value of $f_2(\mathbf{u}_{i,g})$, in such a way that $f_2(\tilde{\mathbf{u}}_{f1,i,g}) \le f_2(\mathbf{u}_{i,g}) + \gamma|f_2(\mathbf{u}_{i,g})|$ for some $\gamma > 0$. This means that the new individual can also slightly worsen

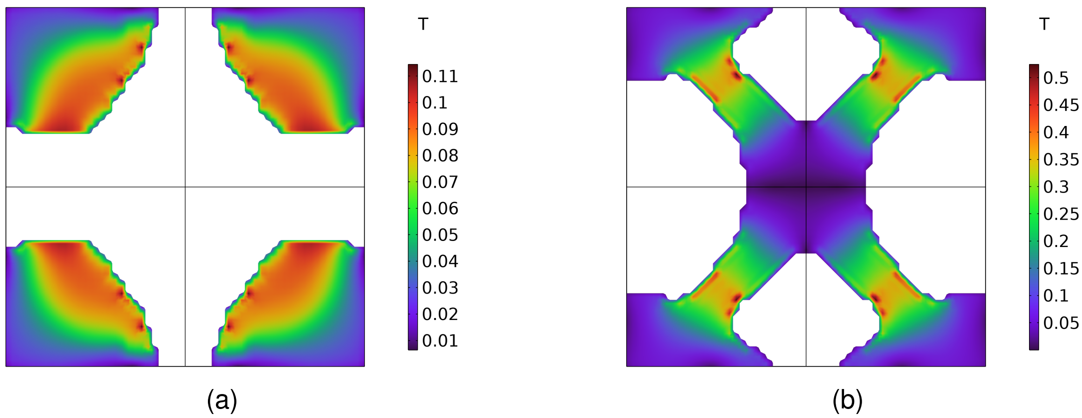

Fig. 5. Top view of the ferrite topologies and magnetic induction field ($\|\mathbf{B}\|$) corresponding to ave. f_1 (a) and ave. f_2 (b) individuals.

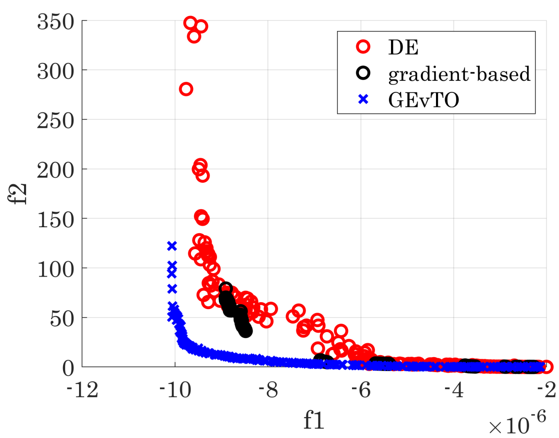

Fig. 6. Comparison of PF computed with a multi-objective DE, a gradient-based procedure, and the GEvTO for the same number of objective evaluations.

the value of f_2.

Similarly, a new individual $\tilde{\mathbf{u}}_{f2,i,g}$ is computed by following the direction of $\mathrm{d}f_2/\mathrm{d}\mathbf{u}_{i,g}$, adding a constraint to $f_1(\tilde{\mathbf{u}}_{f2,i,g})$. The objectives and constraint values are computed for these new individuals and collected into $\mathcal{A}$. The procedure is then repeated for the next generations until the termination criterion is fulfilled. It is worth mentioning that, due to the high computational cost, the gradient search is only performed every g_{grad} iterations.

1) Computational cost: The number of objective function evaluations for the GEvTO is $N_{obj,GEvTO} = N \times g_{max} + 2 \times N \times \lfloor g_{max}/g_{grad} \rfloor$. In comparison, for the DE, $N_{obj,DE} = N \times g_{max}$, while, if only gradient-based steps are used, $N_{obj,grad} = 2 \times N \times g_{max}$.

IV. Results

The GEvTO algorithm is used for optimizing the receiver (RX) ferrite of the WPT device shown in Fig. 3. When analyzing the performance of WPT devices, various metrics have been proposed in the literature. In [20], a multi-objective TO is performed to increase magnetic coupling while minimizing the leakage induction field. In [21], the WS is used to increase the mutual inductance (M) and reduce the leakage field. Single-objective TO are solved in [22], [23].

Here, M and the ferrite core loss P_{core} are considered. From Fig. 3, the mutual inductance between the transmitter (TX) and the RX is computed as:

$$M = \int_{V_{RX}} \mathbf{A} \cdot \hat{\mathbf{t}} \mathrm{d}V \Big/ (S_{RX} I_{TX}) \,, \tag{13}$$

where I_{TX} is the current supplied to the TX coil, S_{RX} is the cross-section area of the RX coil, V_{RX} its volume, and $\hat{\mathbf{t}}$ a versor in the winding direction of the RX coil. Thanks to the symmetry of the problem, only 1/4 of the device is analyzed. P_{core} is computed by integrating over V_{RX} the loss density:

$$p_{core} = k f_{op}^{\alpha} B^{\beta}, \tag{14}$$

where k, α, and β are the Steinmetz parameters, and B denotes the peak magnetic flux density. In this work: $f_{op} = 85$ kHz, $k = 1.97$, $\alpha = 1.5$, and $\beta = 3.22$.

The constrained MOP is defined as:

$$\begin{aligned} \min \quad & \mathbf{f} = [f_1, f_2]^\top = [-M, P_{core}]^\top \\ \text{s.t.} \quad & G(\mathbf{x}) = V(\mathbf{x})/V_{\Omega_d} \leq 0.5 = \bar{G}. \end{aligned} \tag{15}$$

The GEvTO is implemented in MATLAB, while the FEM software COMSOL® Multiphysics 6.3 is used to solve the EM problem, compute the objective functions and their sensitivities. The total time required to solve one EM problem with 163359 unknowns for $\mathbf{y}$ and compute the sensitivities is ≈ 20 s, using a machine with an Intel(R) Xeon(R) w5-2465X (3.10 GHz) and 128 GB of RAM. The number of decision variables is $n = 729$.

Since DE with small populations do not perform well, the population size has been set to $N = 40$, and the maximum archive size is 200. As the termination criterion $g_{max} = 150$ is used. Following [8], the heuristic parameters of the DE are selected as: $\sigma_F = 0.1$ and $CR = 0.5$. The constant c is fixed to 0.1 and $p = 0.2$. While a constant $\beta_{flip} = 0.004$ and $\gamma = 0.02$ are used during the ILP step, which is taken every $g_{grad} = 10$

TABLE I

OBJECTIVE VALUES FOR TWO REPRESENTATIVE TOPOLOGIES.

Topology	$-f_1$	f_2	G
Ave. f_1	6.1 μH	1.7 W	0.49
Ave. f_2	10.1 μH	61.5 W	0.49

generations. Future research activities will discuss the effect of parameters on GEvTO performance, e.g., through CR and β_{flip} adaptation.

The objective space for selected generations ($g = \{5, 30\}$), highlighting feasible and unfeasible individuals, is shown in Fig. 4. At the end of GEvTO iterations a well-resolved convex Pareto Front (PF) is obtained and two representative individuals are extracted, i.e., (a) the individual corresponding to the average f_1 value, and (b) the individual corresponding to the average f_2 value. The topologies are shown in Fig. 5, while the values of objectives and constraints are listed in Table I.

The GEvTO is also compared to a multi-objective DE using the mutation and crossover operators explained in Section III-B, and a gradient-based algorithm exploiting only the SAIP described in Section III-C. The computed PF is shown in Fig. 6. It can be observed that for the same number of objective function evaluations (see SectionIII-D1), the individuals of the DE are dominated by those in the GEvTO archive, while the individuals generated with the SAIP are clustered, and the corresponding PF is disconnected. This confirms the validity of the hybrid strategy proposed in this research.

V. CONCLUSIONS

The article introduces a hybrid evolutionary algorithm with a local gradient-based search for the solution of constrained multi-objective TO problems dealing with binary decision variables. The combination of a global DE search with a filtering strategy and a local gradient search is capable of generating topologies with good connectivity properties. The resulting PF dominates the solutions obtained with a naïve DE and a multi-objective gradient-based algorithm.

Future research will expand the experimental evaluation of the proposed algorithm, assessing its performance on user-defined parameters (e.g., $CR, \gamma, \beta_{flip}, g_{grad}$) by introducing appropriate metrics, and exploiting alternative evolutionary strategies based on a smaller population (e.g., μJADE).

REFERENCES

[1] I. Das and J. E. Dennis, “A closer look at drawbacks of minimizing weighted sums of objectives for Pareto set generation in multicriteria optimization problems,” *Structural optimization*, vol. 14, no. 1, pp. 63–69, 1997.

[2] O. Sigmund, “On the usefulness of non-gradient approaches in topology optimization,” *Structural and Multidisciplinary Optimization*, vol. 43, no. 5, pp. 589–596, 2011.

[3] Y. Hidaka, T. Sato, and H. Igarashi, “Topology Optimization Method Based on On–Off Method and Level Set Approach,” *IEEE Transactions on Magnetics*, vol. 50, no. 2, pp. 617–620, 2014.

[4] M. Bujny, N. Aulig, M. Olhofer, and F. Duddeck, “Hybrid evolutionary approach for level set topology optimization,” in *2016 IEEE Congress on Evolutionary Computation (CEC)*, 2016, pp. 5092–5099.

[5] T. Gauthey, P. Gangl, and M. H. Hassan, “Multi-Material Topology Optimization with Continuous Magnetization Direction for motors design,” in *2022 International Conference on Electrical Machines (ICEM)*, 2022, pp. 483–489.

[6] J. Tucek, M. Capek, L. Jelinek, and O. Sigmund, “Density-Based Topology Optimization in Method of Moments: Q-Factor Minimization,” *IEEE Transactions on Antennas and Propagation*, vol. 71, no. 12, pp. 9738–9751, 2023.

[7] R. Albanese and G. Rubinacci, “Finite Element Methods for the Solution of 3D Eddy Current Problems,” ser. Advances in Imaging and Electron Physics, P. W. Hawkes, Ed. Elsevier, 1997, vol. 102, pp. 1–86.

[8] J. Zhang and A. C. Sanderson, “JADE: Adaptive Differential Evolution With Optional External Archive,” *IEEE Transactions on Evolutionary Computation*, vol. 13, no. 5, pp. 945–958, 2009.

[9] S. Das and P. N. Suganthan, “Differential Evolution: A Survey of the State-of-the-Art,” *IEEE Transactions on Evolutionary Computation*, vol. 15, no. 1, pp. 4–31, 2011.

[10] Y. Li, S. Yang, and Z. Ren, “A Multi-Objective Topology Optimization Methodology Using Deep Learning and Its Application to Electromagnetic Devices,” *IEEE Transactions on Magnetics*, vol. 60, no. 3, pp. 1–5, 2024.

[11] T. Sato, K. Watanabe, and H. Igarashi, “A Topology Optimization of Electromagnetic Devices Based on Kernel Ridge Regression as a Variant of Gaussian Network-Based Shape Representation,” *IEEE Transactions on Magnetics*, vol. 60, no. 12, pp. 1–5, 2024.

[12] F. Guo, N. Tang, and I. P. Brown, “Magneto-Structural Combined Dimensional and Topology Optimization of Interior Permanent Magnet Synchronous Machine Rotors,” *IEEE Transactions on Industry Applications*, vol. 58, no. 6, pp. 7241–7250, 2022.

[13] X. Ma, Y. Zong, P. Zhang, J. Chen, Z. Ren, Z. Liu, and C. Li, “Design of Multiband Patch Antennas Using Adaptive Topology Optimization Method,” *IEEE Transactions on Antennas and Propagation*, pp. 1–1, 2025.

[14] F. Lucchini, R. Torchio, and P. Alotto, “Gradient-Informed Pareto-Based Multi-Objective Binary Topology Optimization,” *IEEE Access*, vol. 13, pp. 86 576–86 587, 2025.

[15] P. A. N. Bosman, “On Gradients and Hybrid Evolutionary Algorithms for Real-Valued Multiobjective Optimization,” *IEEE Transactions on Evolutionary Computation*, vol. 16, no. 1, pp. 51–69, 2012.

[16] K. Sindhya, K. Miettinen, and K. Deb, “A Hybrid Framework for Evolutionary Multi-Objective Optimization,” *IEEE Transactions on Evolutionary Computation*, vol. 17, no. 4, pp. 495–511, 2013.

[17] Y. Liang and G. Cheng, “Review of discrete variable topology optimization by sequential approximate integer programming,” *Engineering Optimization*, vol. 57, no. 1, pp. 130–160, 2025.

[18] R. El Bechari, F. Guyomarch, and S. Brisset, “The Adjoint Variable Method for Computational Electromagnetics,” *Mathematics*, vol. 10, no. 6, 2022.

[19] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, “A fast and elitist multiobjective genetic algorithm: NSGA-II,” *IEEE Transactions on Evolutionary Computation*, vol. 6, no. 2, pp. 182–197, 2002.

[20] Y. Otomo and H. Igarashi, “Topology optimization of magnetic cores for WPT using the geometry projection method,” *COMPEL - The international journal for computation and mathematics in electrical and electronic engineering*, vol. 41, no. 3, pp. 889–899, 10 2021.

[21] M. Hu and U. K. Madawala, “Magnetic Structure Design in IPT Systems Based on Topology Optimization,” *IEEE Transactions on Transportation Electrification*, vol. 11, no. 2, pp. 5374–5386, 2025.

[22] Y. Pei, L. Pichon, Y. L. Bihan, and M. Bensetti, “SIMP-Method Topology Optimization of Ferrite Structures in Inductive Power Transfer Systems,” *IEEE Transactions on Electromagnetic Compatibility*, vol. 65, no. 6, pp. 1744–1750, 2023.

[23] C. Lee, “Topology Optimization of the Transmitter Ferrite and Receiver Coil for Minimizing the Weight of the Wireless-Charging Portable Devices,” *IEEE Transactions on Industrial Electronics*, vol. 71, no. 10, pp. 12 192–12 201, 2024.