Hierarchical Clustering Based on Sequential Preference Stability

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***Abstract*—This paper presents a novel hierarchical clustering methodology grounded in a refined formulation of a sequential preference stability measure. This measure captures the chance that two agents preserve identical opinions on an alternative across consecutive time points, while remaining well-defined in the presence of incomplete preference information. We formalize the measure and derive a decomposition that separates within group and cross group stability contributions for arbitrary agent partitions. Leveraging these theoretical results, we introduce the Stability Hierarchical Clustering Algorithm. An efficient incremental update strategy substantially reduces computational overhead. The methodology is validated by means of two case studies: a synthetic temporal preference profile designed to illustrate its discriminate capability, and an empirical data set concerning life-support decisions of cancer patients, which demonstrates its practical applicability.**

***Index Terms*—Preference stability, temporal preference, hierarchical clustering**

I. Introduction

Clustering is a fundamental unsupervised learning task aimed at discovering latent structure in unlabeled data. Among clustering paradigms, hierarchical clustering remains particularly attractive due to its ability to reveal multilevel organization through a dendrogram, enabling the extraction of consistent partitions at different resolutions without re-executing the algorithm. Since foundational work such as Ward's minimum-variance method [1] and classical hierarchical schemes [2], hierarchical clustering has played a central role in exploratory data analysis.

A defining aspect of hierarchical clustering is its strong dependence on the underlying similarity or dissimilarity measure. As emphasized in both classical and modern surveys, the choice of similarity often has a greater impact on the resulting structure than the specific agglomeration strategy itself [3], [4]. Recent reviews highlight ongoing developments, including scalable implementations, domain-adapted similarity definitions, and hybrid frameworks [5], [6]. From a theoretical perspective, hierarchical clustering has also been formalized in terms of explicit objective functions, further underscoring the central role of similarity definitions [7], [8]. Related directions such as cluster ensemble methods aim to improve robustness by aggregating multiple solutions, but they typically rely on static data representations [9].

Whether preferences remain stable over time is a central issue in intertemporal decision making. Empirical evidence from economics and social choice shows that preference stability varies across contexts and cannot be taken for granted [10]–[12]. These findings motivate the use of explicit measures that capture intertemporal coherence rather than static or single-period agreement.

Temporal preference data introduce additional challenges. In many decision-making contexts, agents express opinions on alternatives across ordered time moments, and these opinions may evolve or be intermittently absent. While substantial work exists on temporal clustering and time series analysis [13], capturing how agents' preferences persist across successive evaluations remains a challenging problem. Measures of preference stability and time cohesiveness have been proposed to quantify such intertemporal consistency [14], [15], but have not been integrated as a guiding criterion for clustering methods.

In this work, we propose a hierarchical clustering framework grounded in a refined sequential preference stability measure, denoted ψ_S. The measure assesses the likelihood that two agents maintain identical opinions on an alternative across consecutive time instants and remains well defined in the presence of missing preference information, a nontrivial issue for the interpretability of temporal preference models [16]. Building on this criterion, we introduce the ψ_S-Stability Hierarchical Clustering Algorithm, which iteratively merges clusters so as to maximize sequential stability and favor groups with coherent temporal behavior.

A key theoretical contribution is an additive decomposition of ψ_S across agent subprofiles, enabling efficient incremental updates of stability scores during agglomeration and substantially reducing computational overhead. The methodology is illustrated using a synthetic temporal preference profile and validated on an empirical data set concerning life-support decisions of cancer patients.

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The main contributions are: (i) the ψ_S measure, allowing missing data; (ii) a hierarchical clustering algorithm based on sequential stability; and (iii) an additive decomposition enabling efficient computation.

The remainder of the paper is organized as follows. Section II introduces the formal framework and the sequential preference stability measure. Section III presents the ψ_S-Stability Hierarchical Clustering Algorithm and its incremental update scheme. Section IV reports the experimental evaluation, and Section V concludes the paper.

Upon acceptance, the authors will provide public access to all data and code associated with this work.

II. Methodology

This section presents the formal framework and notation employed throughout the work. It also introduces our specific formulation of preference stability, referred to as the sequential preference stability measure. Furthermore, the section develops a useful decomposition of this measure across agent subprofiles.

A. Framework and Notation

Let $\mathbf{A} = \{a_1, a_2, ..., a_N\}$ a set of agents or experts. Agents express their opinions on an alternative, x, at an ordered set of time points $\mathbf{T} = \{t_1, \dots, t_T\}$ and let $\mathcal{O} = \{0,\ 0.5,\ 1,\ -\}$ denote the opinion labels on alternative x, where 0 stands for disapproval, 0.5 for undecided, 1 for approval, and $-$ for not expressing an opinion.

Definition 1. [15] A *temporal preference profile*, $\mathbf{P} = \left(P_{it_j}\right)_{N\times T}$, of a set of agents $\mathbf{A}$ on an alternative x at T different time moments is an $N \times T$ matrix, where P_{it_j} is the opinion of the agent i over alternative x at t_j moment.

In particular, an agent may approve the alternative at a given time (indicated as 1 in $\mathbf{P}$), be undecided about it (indicated as 0.5 in $\mathbf{P}$), disapprove it (indicated as 0 in $\mathbf{P}$), or refrain from expressing any opinion at that time (indicated as $-$ in $\mathbf{P}$).

Let $\mathbb{P}_{N\times T}$ denote the set of all such $N \times T$ matrices. For simplicity, $(a)_{N\times T}$ is the $N \times T$ matrix whose cells are universally equal to $a \in \mathbf{O}$. For any temporal preference profile $\mathbf{P}$, the matrix $\mathbf{P}_S$ denotes its restriction to a subset of agents $S \subseteq \mathbf{A}$. This *agent - subprofile* is obtained by selecting from $\mathbf{P}$ the rows corresponding to the agents in S.

Let $S_1, \dots, S_r$ be a partition of $\mathbf{A}$, such that, $S_g \cap S_l = \varnothing$ for all $g \neq l$ in $\{1, \dots, r\}$ and $\mathbf{A} = S_1 \cup \dots \cup S_r$. This partition induces a decomposition of the temporal preference profile $\mathbf{P}$ into agent - subprofiles $\mathbf{P}_{S_1}, \dots, \mathbf{P}_{S_k}$ with $N_1, \dots, N_k$ agents, respectively, $N = N_1 + \dots + N_k$, and whose union reconstructs the original temporal preference profile, namely, $\mathbf{P} = \mathbf{P}_{S_1} \uplus \mathbf{P}_{S_2} \uplus \dots \uplus \mathbf{P}_{S_r}$. This is called an *agent-partition.*

For each temporal preference profile $\mathbf{P}$ on x and for each $t_j \in \mathbf{T}$, let $n_a^{t_j}$ denote the number of agents with opinion $a \in \mathcal{O}$ at time t_j. Hence, the number of agents who express an opinion at t_j is $N_{t_j} = n_0^{t_j} + n_{0.5}^{t_j} + n_1^{t_j}$; when not all agents express an opinion at t_j, we have $N_{t_j} \neq N$.

Times	Disapproval	Undecided	Approval	$-$	Size
$t_1\ t_2$	$N_{0,0}^{t_1,t_2}$	$N_{0.5,0.5}^{t_1,t_2}$	$N_{1,1}^{t_1,t_2}$	$N_{-,-}^{t_1,t_2}$	$\widetilde{N_1}$
	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$t_j\ t_{j+1}$	$N_{0,0}^{t_j,t_{j+1}}$	$N_{0.5,0.5}^{t_j,t_{j+1}}$	$N_{1,1}^{t_j,t_{j+1}}$	$N_{-,-}^{t_j,t_{j+1}}$	$\widetilde{N_j}$
	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$t_{T-1}\ t_T$	$N_{0,0}^{t_{T-1},t_T}$	$N_{0.5,0.5}^{t_{T-1},t_T}$	$N_{1,1}^{t_{T-1},t_T}$	$N_{-,-}^{t_{T-1},t_T}$	$\widetilde{N}_{T-1}$

TABLE I
CONDENSED NOTATION USED IN DEFINITION 2.

For each consecutive pair (t_j, t_{j+1}), we consider the counts $N_{a,b}^{t_j,t_{j+1}}$, $a, b \in \mathcal{O}$, where $N_{a,b}^{t_j,t_{j+1}}$ denotes the number of agents that have the opinion a at t_j and the opinion b at t_{j+1}. In particular, the terms

$$N_{0,0}^{t_j,t_{j+1}}, \quad N_{0.5,0.5}^{t_j,t_{j+1}}, \quad N_{1,1}^{t_j,t_{j+1}}, \quad N_{-,-}^{t_j,t_{j+1}}$$

capture persistence (agents who keep their opinion or absence of opinion). The Table I provides, for each pair $t_j\ t_{j+1}$, these counts together with the total number of agents that express their opinions at times t_j and t_{j+1}, $\widetilde{N}_{t_j t_{j+1}}$, shortly $\widetilde{N_j}$. If all the agents express their opinion at all set of times, then $\tilde{N}_{t_j,t_{j+1}} = N$ for all j. Otherwise, $\tilde{N}_{t_j,t_{j+1}}$ may vary with j due to non response. Note that typically

$$\tilde{N}_{t_j,t_{j+1}} \ \geq\ N_{0,0}^{t_j,t_{j+1}} + N_{0.5,0.5}^{t_j,t_{j+1}} + N_{1,1}^{t_j,t_{j+1}} + N_{-,-}^{t_j,t_{j+1}},$$

with equality if and only if no agent changes their position between t_j and t_{j+1}.

The number of agents who change their opinion between two consecutive instants, t_j and t_{j+1}, from a to b can be represented by $n_{a,b}^{t_j,t_{j+1}}$, for $a \neq b$. And, the total number of changes between t_j and t_{j+1} is

$$\tilde{N}_{t_j,t_{j+1}} - \left(N_{0,0}^{t_j,t_{j+1}} + N_{0.5,0.5}^{t_j,t_{j+1}} + N_{1,1}^{t_j,t_{j+1}} + N_{-,-}^{t_j,t_{j+1}}\right).$$

B. The sequential preference stability measure

Now, we extend a stability measure previously introduced by [15]. A refined formulation is developed termed the sequential preference stability measure.

This extension not only formalizes the temporal dimension of preference consistency, but it is also designed to accommodate instances in which preference information may be missing for certain agents at specific time points, thus allowing the measure to remain well defined in the presence of incomplete preference.

Definition 2. The *sequential preference stability measure* for a group of agents $\mathbf{A}$ on an alternative x at time points $\mathbf{T}$ is the mapping $\psi_S : \mathbb{P}_{N\times T} \to [0,1]$

$$\psi_S(\mathbf{P}) = \frac{1}{T-1} \cdot \sum_{j=1}^{T-1} \left(\sum_{k\in\{0,0.5,1\}} \frac{N_{k,k}^{t_j,t_{j+1}} \cdot (\ N_{k,k}^{t_j,t_{j+1}} - 1)}{\widetilde{N_j}(\widetilde{N_j} - 1)} \right) \quad (1)$$

In an intuitive way, it quantifies the chance that, at a randomly selected point in time, two randomly chosen agents from the group hold the same opinion regarding the given alternative both at that moment and at the immediately subsequent one.

This sequential stability measure is not equivalent to the count of opinion coincidences at the same moments in time. The following example illustrates this distinction.

Example 1. *Consider two agents expressing opinions on alternative x at six times $\mathbf{T} = \{t_1, \ldots, t_6\}$ and the two temporal preference profiles $\mathbf{P}$ and $\mathbf{P}'$*

$$\mathbf{P} = \begin{pmatrix} 0.5 & 0.5 & 1 & 0.5 & 1 & 0.5 \\ 0.5 & 0 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}$$

$$\mathbf{P}' = \begin{pmatrix} 0.5 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0.5 & 0 & 0 \end{pmatrix}$$

Their stability measure are $\psi_S(\mathbf{P}) = 0$ and $\psi_S(\mathbf{P}') = 0.4$ despite both having three coincidences at moments t_2, t_4 and t_6 in $\mathbf{P}$ and t_4, t_5 and t_6 in $\mathbf{P}'$. Hence, the sequential preference stability measure ψ_S captures intertemporal stability rather than the frequency of instantaneous coincidences.

Remark 1. The measure $\psi_S(\mathbf{P})$ attains its extreme values under the following conditions. It holds that $\psi_S(\mathbf{P}) = 1$ iff for every $j = 1, \ldots, T-1$ there exists a single $k \in \{0, 0.5, 1\}$ such that $N_{k,k}^{t_j,t_{j+1}} = \widetilde{N}_j$; that is, all agents retain the same preference value from t_j to t_{j+1}. Conversely, $\psi_S(\mathbf{P}) = 0$ iff for every $j = 1, \ldots, T-1$ and every $k \in \{0, 0.5, 1\}$ we have $N_{k,k}^{t_j,t_{j+1}} \leq 1$, meaning that no preference value is preserved by at least two agents across consecutive time points.

Consequently, the measure lies in the unit interval, with 1 indicating maximal stability and 0 representing its complete absence.

The appealing properties of the sequential preference stability measure set out in [14] can be established for this extended definition without significant difficulty.

The value of ψ_S is sensitive to increases in observable information: substituting missing opinions with explicit evaluations — while keeping previously recorded opinions fixed — may raise or lower the measure depending on whether the newly revealed assessments coincide with those at adjacent time points.

C. Decomposition of the measure across agent-subprofiles

Let $\mathbf{P} = \mathbf{P}_{S_1} \uplus \mathbf{P}_{S_2}$ be a partition of the agent set into two disjoint subprofiles containing N_1 and N_2 agents, respectively, with $N = N_1 + N_2$. Let $\psi_S(\mathbf{P}_{S_1})$ and $\psi_S(\mathbf{P}_{S_2})$ denote the corresponding sequential stability measures. The notation used throughout this decomposition is summarized in Table II.

The proposed sequential preference stability measure admits an additive decomposition into the individual contributions of each agent subprofile and an additional cross-group component term. This decomposition makes it possible to explicitly quantify the marginal contribution of each subprofile to the overall stability.

Times	disapproval	Undecided	Approval	Size
$t_j\ t_{j+1}$	$N_{0,0}^{t_j,t_{j+1}}$	$N_{0.5,0.5}^{t_j,t_{j+1}}$	$N_{1,1}^{t_j,t_{j+1}}$	$\widetilde{N}_j$
$t_j\ t_{j+1}$	$N_{0,0}^{t_j,t_{j+1}}(\mathbf{P}_{S_1})$	$N_{0.5,0.5}^{t_j,t_{j+1}}(\mathbf{P}_{S_1})$	$N_{1,1}^{t_j,t_{j+1}}(\mathbf{P}_{S_1})$	$\widetilde{N}_j(\mathbf{P}_{S_1})$
$t_j\ t_{j+1}$	$N_{0,0}^{t_j,t_{j+1}}(\mathbf{P}_{S_2})$	$N_{0.5,0.5}^{t_j,t_{j+1}}(\mathbf{P}_{S_2})$	$N_{1,1}^{t_j,t_{j+1}}(\mathbf{P}_{S_2})$	$\widetilde{N}_j(\mathbf{P}_{S_2})$

TABLE II
NOTATION USED IN THE AGENT - SUBPROFILE DECOMPOSITION

Although the decomposition is presented for two agent subprofiles the same reasoning extends straightforwardly to agents partitions involving more than two groups.

For each t_j, $j = 1, \ldots, T-1$, and $k \in \{0, 0.5, 1\}$, define

$$Q_k^{t_j} = \frac{N_{k,k}^{t_j,t_{j+1}}\left(N_{k,k}^{t_j,t_{j+1}} - 1\right)}{\widetilde{N}_j(\widetilde{N}_j - 1)}.$$

With this notation, the sequential stability measure (1) can be written as

$$\psi_S(\mathbf{P}) = \frac{1}{T-1} \sum_{j=1}^{T-1} \sum_{k \in \{0,0.5,1\}} Q_k^{t_j}. \tag{2}$$

Since

$$N_{k,k}^{t_j,t_{j+1}} = N_{k,k}^{t_j,t_{j+1}}(\mathbf{P}_{S_1}) + N_{k,k}^{t_j,t_{j+1}}(\mathbf{P}_{S_2}),$$

a direct expansion yields

$$\begin{aligned} N_{k,k}^{t_j,t_{j+1}}\big(N_{k,k}^{t_j,t_{j+1}} - 1\big) = & N_{k,k}^{t_j,t_{j+1}}(\mathbf{P}_{S_1})\big(N_{k,k}^{t_j,t_{j+1}}(\mathbf{P}_{S_1}) - 1\big) \\ & + N_{k,k}^{t_j,t_{j+1}}(\mathbf{P}_{S_2})\big(N_{k,k}^{t_j,t_{j+1}}(\mathbf{P}_{S_2}) - 1\big) \\ & + 2\, N_{k,k}^{t_j,t_{j+1}}(\mathbf{P}_{S_1}) N_{k,k}^{t_j,t_{j+1}}(\mathbf{P}_{S_2}). \end{aligned}$$

Dividing by $\widetilde{N}_j(\widetilde{N}_j - 1)$, we obtain

$$Q_k^{t_j} = w_{1,t_j} Q_k^{t_j}(\mathbf{P}_{S_1}) + w_{2,t_j} Q_k^{t_j}(\mathbf{P}_{S_2}) + \Delta Q_k^{t_j}(\mathbf{P}_{S_1}, \mathbf{P}_{S_2}),$$

where

$$Q_k^{t_j}(\mathbf{P}_{S_i}) = \frac{N_{k,k}^{t_j,t_{j+1}}(\mathbf{P}_{S_i})\left(N_{k,k}^{t_j,t_{j+1}}(\mathbf{P}_{S_i}) - 1\right)}{\widetilde{N}_j(\mathbf{P}_{S_i})\big(\widetilde{N}_j(\mathbf{P}_{S_i}) - 1\big)},$$

$$w_{i,t_j} = \frac{\widetilde{N}_j(\mathbf{P}_{S_i})\big(\widetilde{N}_j(\mathbf{P}_{S_i}) - 1\big)}{\widetilde{N}_j(\widetilde{N}_j - 1)}, \quad i = 1, 2,$$

$$\Delta Q_k^{t_j}(\mathbf{P}_{S_1}, \mathbf{P}_{S_2}) = \frac{2\, N_{k,k}^{t_j,t_{j+1}}(\mathbf{P}_{S_1}) N_{k,k}^{t_j,t_{j+1}}(\mathbf{P}_{S_2})}{\widetilde{N}_j(\widetilde{N}_j - 1)}.$$

Reassembling the terms into (2) yields the following decomposition:

$$\begin{aligned} \psi_S(\mathbf{P}) = \frac{1}{T-1} \sum_{j=1}^{T-1} \sum_{k \in \{0,0.5,1\}} \Big(& w_{1,t_j}\, Q_k^{t_j}(\mathbf{P}_{S_1}) \\ & + w_{2,t_j}\, Q_k^{t_j}(\mathbf{P}_{S_2}) + \Delta Q_k^{t_j}(\mathbf{P}_{S_1}, \mathbf{P}_{S_2}) \Big). \end{aligned} \tag{3}$$

Equation (3) shows that the sequential stability measure decomposes additively into three components: the weighted contributions of each agent subprofile and an extra term accounting for cross-subprofile stability.

The weights w_{1,t_j} and w_{2,t_j} depend solely on the number of active agents at time t_j and represent the proportion of agent pairs contributing within each subprofile.

The cross-term $\Delta Q_k^{t_j}(P_1, P_2)$ captures the contribution of pairs formed by agents belonging to different subprofiles, thus explicitly quantifying inter-group temporal coherence.

This additive structure enables efficient incremental updates of ψ_S during agglomeration, since merging two clusters only requires updating their individual and cross-subprofile contributions. This property is exploited in the hierarchical clustering algorithm introduced in the next section.

III. New Stability-Based Hierarchical Clustering Algorithm

This section presents the proposed ψ_S - Stability Hierarchical Clustering Algorithm, which provides an agglomerative clustering procedure grounded on the notion of sequential preference stability. The method replaces traditional distance- or linkage-based criteria with a stability-driven merging strategy tailored to temporal preference data.

The algorithm is initialized by considering each agent as an individual cluster. At each iteration, all pairwise unions of clusters are evaluated by means of the sequential preference stability measure, and the pair whose union attains the highest stability value is selected for merging. This process continues until all agents are merged into a single cluster. The detailed operational steps are presented in Algorithm 1.

Algorithm 1 ψ_S-Stability Hierarchical Clustering Algorithm

Input: $\mathbf{P}$: A temporal preference profile
Output: hc : hierarchical clustering object
Step 0: Initialize clusters
  $\mathcal{C} \leftarrow \{C_i = \{(P_{i1}, \ldots, P_{iT})\} \mid i = 1, \ldots, N\}$
Compute initial sequential stability measures $S \in \mathbb{R}^{|\mathcal{C}|\times|\mathcal{C}|}$ where $S_{ij} \leftarrow \psi_S(C_i \uplus C_j)$ for all $C_i, C_j \in \mathcal{C}$, $i < j$
while $|\mathcal{C}| > 1$ **do**
  Step 1: Create new cluster;
  Find $(p, q) \leftarrow \arg\max_{i<j} S_{ij}$
  Let $C_{\text{new}} \leftarrow C_p \cup C_q$, $N_{\text{new}} \leftarrow |C_p| + |C_q|$
  Record merge (C_p, C_q) at height S_{pq} (for dendrogram)
  Update $\mathcal{C}$: Remove C_p and C_q from $\mathcal{C}$. Add C_{new} to $\mathcal{C}$
  Step 2: Update stability measure matrix:
  for each $C_k \in \mathcal{C} \setminus \{C_{\text{new}}\}$ **do**
    $S_{k\,\text{new}} \leftarrow \psi_S(\mathbf{P}_{\mathcal{C}_k} \uplus \mathbf{P}_{\mathcal{C}_{new}})$
  end for
end while
Return hc: dendrogram encoding all merges

The decomposition introduced in II-C facilitates the incremental updating of all sequential preference stability measures, thereby eliminating the need for repeated passes over the original temporal preference profile. This strategy leads to a substantial reduction in constant factors in the running time, while preserving the asymptotic complexity of the algorithm.

IV. Experimental Evaluation

This section demonstrates the applicability of the proposed hierarchical clustering method through two complementary case studies. The first involves a synthetic temporal preference profile specifically constructed to assess the model's internal logic and discriminative power. The second applies the algorithm to a previously published case, illustrating the practical effectiveness in a real-world scenario.

A. Case Study: synthetic data

An artificial temporal preference profile is constructed for a set of twelve agents, each of whom may express an opinion on a given alternative x across six distinct time points, denoted by $\mathbf{T} = \{t_1, \ldots, t_6\}$. The agents are organized into three well-defined groups. The resulting profile is given in Table III. The Table IV presents the intermediate computations leading to the value $\psi(P) = 0.2097$.

	t_1	t_2	t_3	t_4	t_5	t_6
Agent 1	1	1	1	0	0	0
Agent 2	1	1	1	0	1	0
Agent 3	1	1	1	1	0	0
Agent 4	1	1	1	0	0	1
Agent 5	0	0	0	1	1	1
Agent 6	0	–	0	1	1	1
Agent 7	1	0	0	1	1	1
Agent 8	–	0.5	0.5	0.5	0.5	1
Agent 9	0	0.5	0.5	0.5	1	1
Agent 10	0	0.5	0.5	0.5	1	1
Agent 11	–	–	0.5	0.5	1	1
Agent 12	–	0.5	0.5	0.5	1	1

TABLE III
SYNTHETIC TEMPORAL PROFILE

Times	$N_{0\,0}$	$N_{0.5\,0.5}$	$N_{1\,1}$	$N_{-\,-}$	$\widetilde{N_j}$
t_1 t_2	1	0	4	1	8
t_2 t_3	2	4	4	0	10
t_3 t_4	0	5	1	0	12
t_4 t_5	2	1	3	0	12
t_5 t_6	2	0	7	0	12

TABLE IV
INPUTS FOR COMPUTING ψ_S FOR THE SYNTHETIC PROFILE

After applying the ψ_S-Stability Hierarchical Clustering Algorithm, the corresponding dendrogram was generated, as shown in Fig. 1.

In hierarchical clustering, the selection of the cut level in the dendrogram determines the number of clusters in the resulting partition. In this case, the cut for three groups applied to the structure in Fig. 1 accurately recovers the three groups embedded in the synthetic profile. The final cluster assignments are summarized in Table V.

Furthermore, Fig. 2 provides a combined visualization of the dendrogram and the heatmap of the pairwise sequential preference stability matrix. This representation serves as a diagnostic tool that facilitates the assessment of both the hierarchical structure produced by the clustering algorithm and

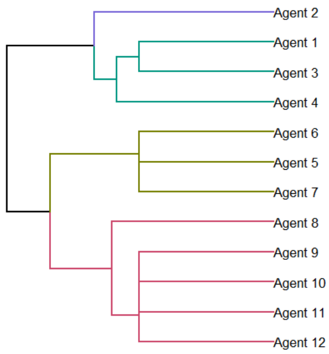

Fig. 1. Dendrogram derived from the synthetic profile in Table III. The clustering reveals well-defined groups.

the underlying pairwise stability relationships. In the present case, it offers additional evidence supporting the separability of the groups, as shown in the accompanying plot.

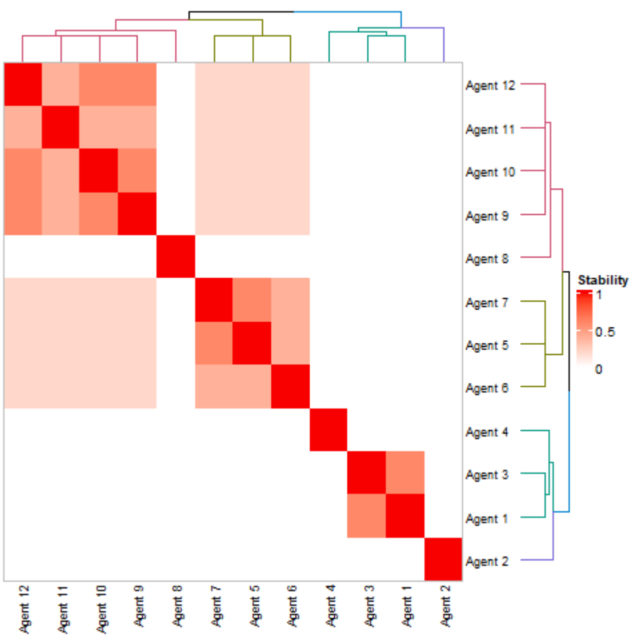

Fig. 2. Heatmap of the stability matrix and associated dendrogram for synthetic profile in Table III. Higher color intensity indicates stronger pairwise stability. The clustering structure is consistent with the heatmap, revealing well-defined groups with clear separation between clusters.

Cluster	Agents	Cluster size	$\psi(P_S)$
1	1,2, 3, 4	4	0.4667
2	5, 6, 7	3	0.6000
3	8, 9, 10, 11, 12	5	0.5200

TABLE V
SUMMARY OF CLASSIFICATION IN THREE CLUSTERS FOR SYNTHETIC PROFILE IN TABLE III

B. Case study 2: Cancer Patients' Life Support Preferences

In this section, a case study is presented to illustrate the performance of the hierarchical clustering algorithm in an empirical scenario. The data consist of cancer patients' preferences regarding life - support decisions, previously regarded by [14].

The study involved questionnaires of 257 patients who expressed their preferences about approving or disapproving various treatments for life-sustaining at end of life at four different time moments along their illness. One of those treatments, dying in an intensive care unit (ICU), is considered here. Thus, patients' opinions can be formalized by means of a temporal preference profile $\mathbf{P}^{ICU} = \left(P^{ICU}_{it_1}\right)_{257\times 4}$

The Table VI contains the distinct rows in the profile along with the number of times each one is repeated. The Table VII condenses the relevant counts to calculate the sequential preference stability measure $\psi(\mathbf{P}^{ICU}) = 0.449$. The sequential preference stability indicates moderate temporal consistency, with some heterogeneity in patients' opinion trajectories, motivating the use of clustering to identify more homogeneous subgroups.

	t_1	t_2	t_3	t_4	count
Type 1	0	0	0	0	142
Type 2	0	1	0	0	14
Type 3	1	0	0	0	14
Type 4	1	0	1	0	8
Type 5	1	1	0	0	14
Type 6	1	1	0	1	2
Type 7	1	1	1	0	37
Type 8	1	1	1	1	26

TABLE VI
TEMPORAL PROFILE AFTER GROUPING THE REPEATED ROWS IN $\mathbf{P}^{ICU}$

Times	$N_{0\,0}$	$N_{0.5\,0.5}$	$N_{1\,1}$
t_1 t_2	142	0	79
t_2 t_3	156	0	63
t_3 t_4	184	0	26

TABLE VII
NUMBER OF PATIENTS THAT APPROVE ($N_{0\,0}$) AND DISAPPROVE ($N_{1\,1}$) ICU TREATMENT AT DIFFERENT MOMENTS OF TIME IN PROFILE $\mathbf{P}^{ICU}$

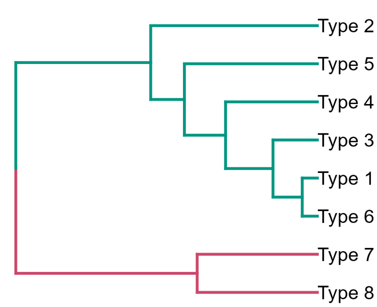

Fig. 3. Dendrogram derived from profile $\mathbf{P}^{ICU}$, highlighting the hierarchical relationships. The clustering reveals two well-defined groups , indicating strong intra-group similarity and clear separation between clusters.

The ψ_S-Stability Hierarchical Clustering Algorithm was applied to this profile. The resulting dendrogram is shown in Fig. 4. Two well-defined clusters are clearly identifiable that are summarized in the Table VIII .

Cluster	Members	Cluster size	$\psi(P_S)$
1	Type 1 to type 6	194	0.6954
2	Type 7 and type 8	63	0.7803

TABLE VIII
SUMMARY OF CLASSIFICATION IN TWO CLUSTERS OF THE PROFILE $\mathbf{P}^{ICU}$

Fig. 3 presents the combined display of the dendrogram and the heatmap corresponding to the pairwise sequential preference stability measures. This integrated view enhances the interpretability.

The two clusters identified by the ψ_S-Stability Hierarchical Clustering Algorithm reflect distinct preference dynamics. Cluster 1 is associated with increasing rejection of ICU admission, while Cluster 2 reflects stable support for life-sustaining interventions.

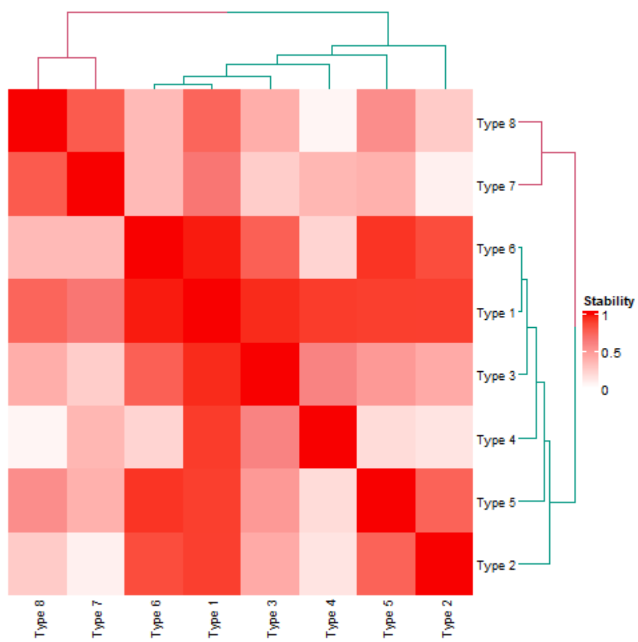

Fig. 4. Heatmap of the stability matrix and associated dendrogram for profile $\mathbf{P}^{ICU}$. Higher color intensity indicates stronger pairwise stability between types. The clustering structure is consistent with the heatmap.

From an applied perspective, these results suggest that cancer patients' preferences regarding ICU care are not only heterogeneous but also structured around distinct and stable temporal trajectories. Identifying such groups may support advance care planning by distinguishing stable and disease-sensitive decisions.

V. FINAL REMARKS

This paper has presented a hierarchical clustering methodology grounded in a refined formulation of a sequential preference stability measure. The proposed measure captures the likelihood that two agents preserve identical opinions on an alternative across consecutive time points and remains well defined in the presence of incomplete preference information. Its formalization and additive decomposition into within-group and cross-group contributions provide a clear and interpretable characterization of temporal preference stability.

Building on these theoretical results, the ψ-Stability Hierarchical Clustering Algorithm introduces a novel agglomerative criterion based on temporal preference behavior. The derived decomposition enables efficient incremental updates during agglomeration, reducing computational overhead while preserving the asymptotic complexity of the clustering process.

The proposed methodology has been validated through two case studies: a synthetic temporal preference profile designed to illustrate its discriminative capability, and an empirical data set concerning life-support decisions of cancer patients, which demonstrates its practical applicability.

Future research directions include the adaptation of internal clustering validation indices to stability-based frameworks, the exploration of alternative formulations of temporal coherence, and the extension of the approach to more complex temporal preference settings.

[1] The authors provide public access to all data and code associated with this work in DOI: 10.5281/zenodo.19578791

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