DPC-HOPE: A Fuzzy Density-Peak Approach to Overlapping Community Detection

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***Abstract*—Overlapping community detection in complex networks remains a fundamental unresolved problem due to the fuzzy nature of node memberships and the computational bottleneck imposed by the $O(n^2)$ complexity of conventional density-peak algorithms. Real-world networks are characterized by continuous membership distributions in which nodes belong to multiple functional groups. However, existing methods often compromise this structural realism by imposing discrete assignments or are computationally infeasible for networks at the million-node scale. In this paper, we propose DPC-HOPE, a framework that uses Density Peak Clustering (DPC) using a Structural Fuzzy Membership Propagation lens. We replace Euclidean distance with a High-Order Proximity Embedding (HOPE) and then introduce a Fuzzy Directed Acyclic Graph (F-DAG) propagation mechanism to model the gradient of membership flow from community peaks to the boundaries. This enables an explicit, non-heuristic determination of overlapping memberships with a computational complexity of $O(m + n \log n)$. Experimental results on large-scale LFR benchmarks and real-world networks demonstrate that DPC-HOPE consistently outperforms existing community detection algorithms in both accuracy and community quality. These results suggest that explicitly capturing topological fuzziness is essential to achieving both scalability and accuracy in large-scale network analysis.**

***Index Terms*—Overlapping Community Detection, Density Peak clustering, High-Order Proximity (HOPE), Fuzzy Directed Acyclic Graph, Structural Graph Embedding, Fuzzy Community Detection, Complex Network Analysis**

I. Introduction

As network science continues to evolve, uncovering latent organizational structures within large-scale, complex datasets has emerged as a central objective of contemporary data mining. Real-world networks, ranging from social, technological, and biological systems [1], are not disjoint but exhibit structural ambiguity, in which nodes often maintain simultaneous memberships in multiple functional communities [2]. While community detection is extensively researched, the shift from discrete network partitioning towards modelling overlapping and fuzzy network structures has exposed a fundamental scalability–precision trade-off. Recent works in the literature are majorly divided into two parts: 1) Fuzzy-based approaches that deliver high-resolution community assignments but incur $O(n^2)$ or $O(n^3)$ computational complexity, which makes them impractical for large-scale networks, 2) Heuristic-based overlapping methods that scale efficiently but impose hard boundaries incapable of capturing nuanced membership gradients in transition zones [1]. DPC-HOPE is motivated by the observation that community membership is not inherently binary, but rather exhibits a continuous topological flow [3]. In real-world networks, the influence of a community core weakens as we move towards its boundary. Although traditional Density Peak Clustering [4] effectively identifies these cores, it neglects the gradual decay of influence and instead imposes hard, winner-take-all assignments. This motivates the need for a framework that integrates DPC's intuitive peak detection with a fuzzy Membership Propagation model capable of capturing gradual membership transitions at scale. In this paper, we introduce DPC-HOPE, which integrates the logic of Density Peak Clustering (DPC) with the representational power of High-Order Proximity Embeddings (HOPE) [5]. We shift from discrete clustering to a continuous, Fuzzy Directed Acyclic Graph (F-DAG) propagation model. Unlike methods that extend DPC, which have a complexity of $O(n^2)$, DPC-HOPE has a computational complexity of $O(m + n \log n)$, making it one of the few fuzzy-based frameworks which can process large-scale networks with high accuracy. In this paper, we bridge this gap by proposing the DPC-HOPE framework. Our contributions are as follows:

- **Novel Fuzzy-Topological Synthesis:** We present a framework that reconceptualizes community detection as a structural fuzzy Membership Propagation problem by combining High-Order Proximity Embeddings (HOPE) with Density Peak Clustering (DPC), enabling the detailed modelling of the structure beyond first-order connectivity.
- **F-DAG Propagation Mechanism:** We propose the Fuzzy Directed Acyclic Graph (F-DAG), a membership propagation model. F-DAG enables nodes to aggregate membership degrees from multiple peaks, thereby supporting a non-iterative computation of multi-community affiliation. By treating community centers as density "peaks", we let the membership "flow" toward community boundaries as a fuzzy transition, which enables non-heuristic determination of node belongingness.
- **Linear-Logarithmic Scalability:** By combining hierarchical proximity indexing (HNSW) with local push–flow dynamics, we achieve an $O(m+n \log n)$ time complexity for the full fuzzy Membership Propagation process, effec-

tively overcoming the quadratic limitations of traditional density-peak and fuzzy community detection methods.

We perform an extensive comparative study of large-scale real-world networks such as YouTube and GitHub, as well as LFR benchmarks with up to 10^6 nodes. Our results show that DPC-HOPE consistently achieves higher accuracy and Modularity scores. The results show that explicitly modelling structural fuzziness is essential for accurately capturing the decentralized and overlapping architectures of contemporary digital networks [6]. The paper is structured as follows: Section II reviews the literature relevant to the proposed approach. Section III presents the preliminaries of the proposed framework. Section IV details the methodology of the proposed framework. Section V presents the experimental setup and followed by an analysis of the results. Finally, Section VI concludes the paper and discusses potential future research directions.

II. Related Works

Community detection research has steadily advanced across multiple paradigms, revealing a systematic shift from hard, disjoint partitions toward flexible methods that model overlapping community structures.

A. Traditional Disjoint Approaches

Early research in community detection was largely driven by the optimization of modularity. The Louvain [7] algorithm and its successor, the Leiden [8] algorithm widely adopted standards due to their computational efficiency and scalability to large graphs. Despite these advantages, both approaches suffer from the resolution limit, leading to the unintended merging of small, well-defined communities in massive graphs. Moreover, their disjoint formulation assumes exclusive community membership, contradicting the overlapping nature of real-world social and biological systems.

B. Overlapping and Label Propagation-based approaches

In algorithms based on label propagation, including COPRA [9], SLPA [10] and LaProFC [11], nodes iteratively update their labels based on neighbors to achieve consensus. Despite their computational efficiency, these methods are inherently stochastic, producing variable results across different runs. Hence, the observed "fuzziness" in overlapping communities often reflects procedural randomness rather than underlying structural properties. Moreover, due to a lack of a formal objective function, outputs are sensitive to initialization and result in noisy overlapping structures.

C. Higher-Order Structural and Motif-Based Approaches

Higher-order methods move beyond edges to examine small subgraphs or motifs to overcome the limitations of edge-centric models. Methods like CPM [2] and RaidB [12] treats communities as sequences of overlapping k-cliques, whereas Benson et al. introduced motif-based clustering that uses motif conductance to capture functional units such as triangles and feed-forward loops. EdMot [13] is a recent example of this approach. Despite their success, these methods are computationally expensive, typically $O(n^3)$ or higher complexity due to k-clique enumeration. They also suffer from fragmentation, as nodes outside targeted motifs frequently remain unassigned.

D. Density Peak Clustering (DPC) based approaches

The traditional DPC identifies communities by locating structurally separated density peaks. When extended to graphs, metrics like Jaccard similarity or shortest paths are used to measure distance. However, DPC [4] costs $O(n^2)$ due to its all-pairs computation, and overlapping DPC extensions depend on heuristic expansion procedures that often fail to capture the continuous, fuzzy membership gradients of nodes located near community boundaries. DPC-HOPE is designed to address the challenges faced by existing community detection approaches. Unlike motif-based approaches, which require costly subgraph enumeration, it models higher-order proximity implicitly through HOPE embeddings and PageRank-based density, thus capturing network structure without the $O(n^3)$ cost of explicit motif counting. By employing Fuzzy-DAG (F-DAG) propagation, it replaces stochastic label propagation with a deterministic mechanism, while the structural density ρ, ensures that every node is naturally integrated into the fuzzy membership manifold. This achieves $O(m + n \log n)$ computational complexity while preserving the granular accuracy of higher-order network analysis.

III. Preliminaries

Let $G = (V, E)$ be an unweighted, undirected graph where $V = \{v_1, v_2, \dots, v_n\}$ is the set of nodes and $E \subseteq V \times V$ is the set of edges.

Definition 1 (Structural Density ρ): We define density ρ_i for each node $v_i \in V$ based on the structural importance of that node in the network. Rather than relying on local degree density, we leverage the stationary distribution of a personalized random walk to capture higher-order structural proximity.

$$\rho_i = PR(v_i, G) \tag{1}$$

The PageRank score PR captures structural centrality, assigning higher density to nodes at the heart of its community than the noisy nodes in the boundary.

Definition 2 (Relative Distance δ): For each node v_i, δ_i represents the smallest structural distance to any node v_j with a higher density ($\rho_j > \rho_i$):

$$\delta_i = \min_{j:\rho_j > \rho_i} (d_{struc}(i, j)) \tag{2}$$

For the node with the highest density, $\delta_{max} = \max_i(\delta_i)$. Nodes with unusually high ρ and δ are identified as Community Peaks ($\mathcal{P}$).

Definition 3 (Fuzzy Membership Vector U): The membership of node v_i is a vector $\mathbf{u}_i = [u_{i,1}, u_{i,2}, \dots, u_{i,K}]$, where $u_{i,k} \in [0, 1]$ represents the degree of belonging to community k, satisfying $\sum_{k=1}^{K} u_{i,k} = 1$.

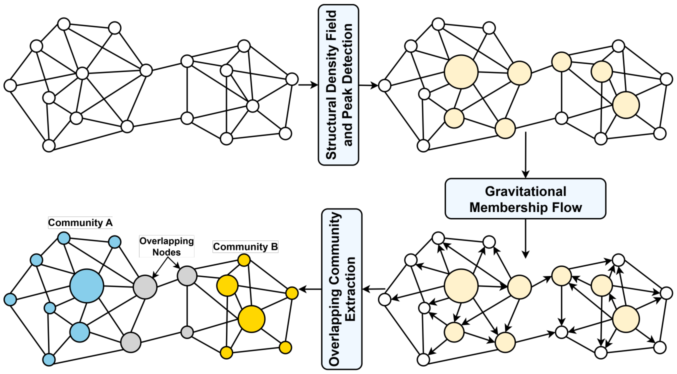

Fig. 1. Flowchart of the proposed algorithm.

IV. Proposed Approach

DPC-HOPE is designed to treat community membership as a topological gradient. We model the network as a structural landscape, where community peaks denote cores and the surrounding valleys capture the fuzzy boundaries of overlapping regions. Fig. 1 explains the flow of the proposed approach.

A. Defining Structural Altitude

We begin with an assessment of node importance, rather than relying on local degree, we adopt a High-Order Proximity Embedding (HOPE) approach. We define the structural density ($\boldsymbol{\rho}$) of a node i by its influence within the global flow of the network, captured through the stationary distribution of a personalized random walk (PageRank).

$$\boldsymbol{\rho} = (1-\alpha)\mathbf{v} + \alpha P^{\top}\boldsymbol{\rho} \tag{3}$$

Where $\mathbf{v}$ is the teleportation vector, $P = D^{-1}A$ is the row-normalized transition matrix and $\alpha \in [0,1]$ is the damping factor.

This ensures that hubs at the centers of dense clusters emerge as true peaks, whereas bridge nodes occupying inter-community valleys receive lower values.

B. Identifying Density Peaks

Once we defined the node altitude, we proceed to identify the mountain peaks, the true centers of communities. These are nodes with high density and are at a large distance from a node with higher density than their own, i.e, structurally isolated from other high-density areas. We compute the relative proximity (δ) for each node, defined as the minimum structural distance to any node j such that $\rho_j > \rho_i$. When ρ and δ are plotted together in a decision graph, the true structural community centers reveal themselves as outliers in the top-right quadrant, enabling automatic determination of the number of communities without manual intervention.

C. Fuzzy-DAG

At this stage, the framework transitions from community peak identification to membership propagation. To enable fuzziness, we introduce a Fuzzy Directed Acyclic Graph (F-DAG), a hierarchy of influence rather than a traditional graph.

Algorithm 1: DPC-HOPE Framework

DPC-HOPE: Density Peak Clustering via High-Order Proximity

Input : Graph $G = (V, E)$, Damping factor α, Proximity threshold d_c, Membership threshold τ

Output: Fuzzy Membership Matrix U

// Phase 1: Structural Altitude (Density Estimation)
Compute High-Order Proximity via Power Iteration: $\rho \leftarrow (1-\alpha)\mathbf{e} + \alpha P^{\top}\rho$;
Normalize densities $\rho_i \in [0,1]$ for scale-invariance;
// Phase 2: Structural Isolation (Distance Calculation)
for *each node* $i \in V$ **do**
  if $\rho_i = \max(\rho)$ **then**
    $\delta_i \leftarrow \max_j(\text{dist}(i,j))$;
  else
    $\delta_i \leftarrow \min_{j:\rho_j > \rho_i}(\text{dist}(i,j))$
    //dist calculated in HOPE embedding space
// Phase 3: Community Center Identification
Calculate significance score $\gamma_i \leftarrow \rho_i \times \delta_i, \forall i \in V$;
Identify top K outliers above the distribution elbow as centers $\mathcal{C} = \{c_1, \dots, c_k\}$;
for *each center* $c_k \in \mathcal{C}$ **do**
  Initialize one-hot membership: $u_{c_k,k} \leftarrow 1.0$;
// Phase 4: F-DAG Construction and Fuzzy Propagation
Define Directed Edges $E_f = \{j \to i \mid (i,j) \in E \wedge \rho_j > \rho_i\}$;
Sort non-center nodes $V \setminus \mathcal{C}$ in descending order of ρ;
for *each node* i *in sorted order* **do**
  Identify Parent set $P_i = \{j \mid (j \to i) \in E_f\}$;
  Calculate inheritance weights: $w_{j,i} = \frac{\rho_j}{\sum_{m \in P_i} \rho_m}$;
  Update membership vector: $\mathbf{u}_i \leftarrow \sum_{j \in P_i} w_{j,i} \cdot \mathbf{u}_j$;
// Phase 5: Thresholding and Overlap Extraction
for *each* $u_{i,k} \in U$ **do**
  if $u_{i,k} > \tau$ **then**
    Assign node i to community k;
return U;

Directed edges flow from high-density nodes to their lower-density neighbors, creating a gravity-led system in which community identity cascades downward from the peaks. For each non-peak node i, its membership vector $\mathbf{u}_i$ is not directly assigned but inherited: node i aggregates membership contributions from all of its parent nodes in the F-DAG, i.e. from neighbors with higher density using a weighted summation.

$$u_{i,k} = \sum_{j \in Parents(i)} \left(\frac{\rho_j}{\sum_{m \in Parents(i)} \rho_m} \right) \cdot u_{j,k} \tag{4}$$

If a node resides between two or more peaks, it inherits membership from each, resulting in a fuzzy assignment. In this

TABLE I
SPECIFIC PARAMETERS FOR LFR BENCHMARK GENERATION

Parameter Description	Notation	Value
Number of nodes	N	10^3 to 10^6
Mixing parameter	μ	$[0.1, 0.7]$
Fraction of overlapping nodes	on	$10\% - 50\%$
Overlapping memberships	om	$[2, 10]$
Average degree	k	15
Maximum degree	$maxk$	50
Degree exponent	τ_1	2
Community exponent	τ_2	1
Min community size	$minc$	20
Max community size	$maxc$	100

TABLE II
STATISTICS OF REAL-WORLD DATASETS

Dataset	Acronym	Nodes	Edges
Autonomous System	AS	6,474	13,895
Github-Musae	GM	37,700	2,89,003
YouTube	YT	11,34,890	29,87,624

step, the membership value u_{ik} is continuous and quantifies the probability of node i belonging to community k.

D. Adaptive overlap thresholding

In this phase of the methodology, fuzzy membership vectors are converted into explicit community lists using an adaptive threshold τ. Node i is assigned to community k if and only if $u_{i,k} > \tau$. Given the normalization constraint $\sum u_i = 1$, higher threshold values yield effectively disjoint community structures, whereas lower thresholds $(e.g., \tau = 0.2)$ enable the identification of overlapping memberships, thereby capturing the multifunctional roles that nodes assume in real-world networks.

E. Time and Space Complexity Analysis

In this section, we will derive the time complexity of the proposed DPC-HOPE. The power iteration for PageRank converges in $O(m)$ for sparse graphs. By using HNSW (Hierarchical Navigable Small World) indexing for proximity lookups, we reduce the search for δ from $O(n^2)$ to $O(n \log n)$. Since the F-DAG is built on the existing edges and processed in a single pass (descending ρ), the assignment phase is strictly $O(m + n)$. Thus, the total time complexity of DPC-HOPE is $O(m + n \log n)$. Notably, DPC-HOPE achieves a linear-logarithmic runtime, effectively breaking the quadratic bottleneck of the original DPC and establishing a new benchmark for scalable fuzzy community detection.

V. EXPERIMENTAL RESULTS AND ANALYSIS

To validate the performance and robustness of the DPC-HOPE framework, we conduct a comprehensive empirical evaluation on both synthetic benchmarks and large-scale real-world networks. Our analysis emphasizes four key dimensions: scalability, robustness to noise, effectiveness in handling overlapping structures, and structural validity.

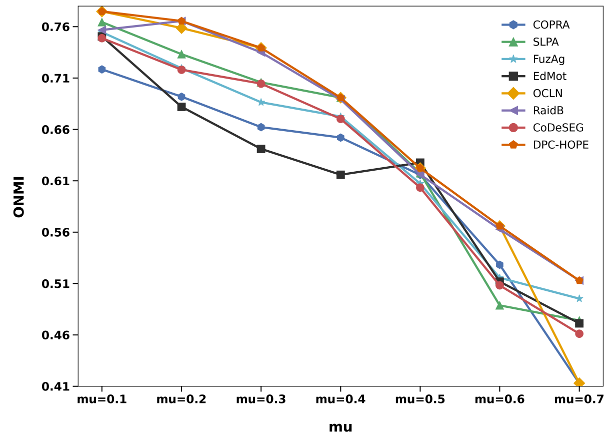

Fig. 2. Plot of the performance of algorithms with increasing noise.

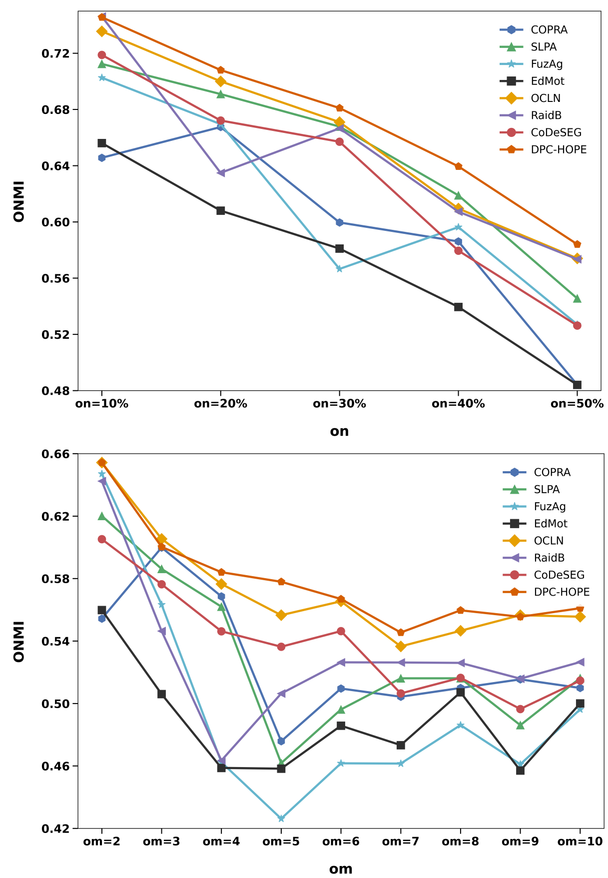

Fig. 3. Plot depicting the performance of algorithms with increasing network overlap.

A. Experimental Setup

We use the LFR (Lancichinetti-Fortunato-Radicchi) [14] benchmark to generate networks with known ground-truth communities. The parameter values used to generate the datasets are given in Table I. We evaluate across three diverse real-world networks: Autonomous systems, GitHub, and YouTube. The details of these datasets are given in Table II.

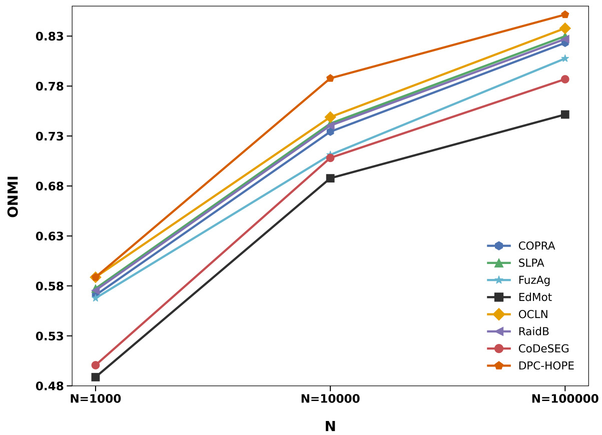

Fig. 4. Plot showing the performance of the algorithms with increasing network size.

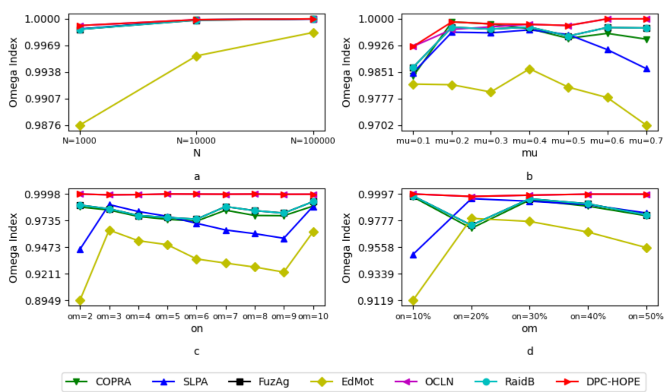

Fig. 5. Comparison of performance of the algorithms with varying network structure evaluated using Omega Index.

We compare the proposed DPC-HOPE against state-of-the-art overlapping methods: COPRA [9], SLPA [10], FuzAg [15], EdMot [13], OCLN [16], RaidB [12] and CoDeSEG [17]. For accuracy evaluation on synthetic datasets where ground-truth is available, ONMI [18] and the Omega Index [19] is employed. To quantify quality, eQ [6] (Extended Modularity) and Soft Modularity [20] are used for real-world networks.

B. Robustness to network noise

The mixing parameter μ specifies the fraction of edges connecting a node to communities other than its own, higher values of μ introduce greater noise and increasingly blurred community structure. As we can observe from fig. 2, as μ increases from 0.1 to 0.7, the performance of algorithms declines drastically. However, DPC-HOPE performs better than its competitors under extreme network conditions, such as 0.7 in ONMI, and maintains a high Omega Index. Since DPC-HOPE identifies community centers as Density Peaks, even under highly noisy conditions ($\mu = 0.7$), the cores of communities remain structurally prominent. Since our method identifies peaks before propagating memberships outward, it

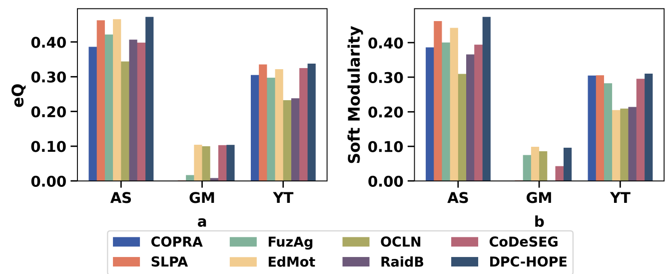

Fig. 6. Performance of the algorithm on large-scale real datasets.

avoids the boundary blurring that leads Label Propagation and Spectral approaches to merge separate communities.

C. Accuracy in handling overlaps

We evaluated the proposed approach under varying levels of overlap intensity by increasing both the proportion of overlapping nodes (o_n) and the number of communities per node (o_m). As shown in fig. 3, in experiments measuring ONMI across networks with varying o_n, we observe that the proposed DPC-HOPE consistently outperforms its competitors. In the OM vs. ONMI tests, DPC-HOPE remains the most stable, and its omega values stay close to 1. This robustness arises from DPC-HOPE's explicit fuzzy gradient: by modelling membership as a topological flow inherited from multiple peaks, the approach naturally captures the shared roles of bridge nodes, something label-voting methods such as SLPA struggle to achieve under high overlap.

D. Performance with increasing network size

As the network size increases from $1,000$ to $100,000$ nodes, DPC-HOPE exhibits exceptional stability in community detection accuracy. From the fig. 4, we can observe that DPC-HOPE consistently outperforms all competitors. The algorithm's advantage arises from the combination of HOPE embeddings, which capture structural information consistently across scales, and deterministic Fuzzy-DAG propagation, which conveys community identity from peaks to all nodes. This approach avoids the instability of label propagation and mitigates the resolution limit that affects modularity-based methods like EdMot.

E. Performance on Omega Index

The performance of the proposed approach and its competitors, evaluated using the omega index, is given in Figure 5. As shown, the proposed approach consistently outperforms its competitors across varying network sizes, densities, overlaps, and complexities.

F. Performance on large-scale real networks

DPC-HOPE demonstrates superior performance in both Extended Modularity and Soft Modularity across all tested networks, as shown in fig. 6. This performance is due to the dataset's hub-and-spoke architecture. Networks such as YouTube and GitHub often follow hub-and-spoke patterns.

The Decision Graph strategy in DPC-HOPE is specifically designed to leverage this topology, in which high-degree hubs are identified as density peaks, while the surrounding nodes constitute the community mass. This structural modelling enables the algorithm to maximize modularity in a deterministic, topology-aware manner, outperforming heuristic-based methods.

G. Limitations and future directions

While the proposed framework establishes a new standard in fuzzy community detection with exceptional robustness and better performance, there is scope for innovation and expansion. The proposed DPC-HOPE is designed for static graphs, it can be extended to detect communities in dynamic networks. While the algorithm performs well on a single-layer network, an extension of the work could be to extend it to a multilayer network.

VI. Conclusion

Overlapping community detection at scale remains one of the most fundamental challenges in contemporary network research. We propose DPC-HOPE, a framework that integrates the local density-based precision of Density Peak Clustering with the global structural modelling capabilities of High-Order Proximity embeddings. Extensive testing confirms that DPC-HOPE is a paradigm shift in overlapping community detection. By consistently surpassing state-of-the-art methods such as SLPA, OCLN, and EdMot on both synthetic and real-world networks, the framework proves that Fuzzy-DAG propagation captures the subtle, multi-dimensional memberships characteristic of complex systems. Remarkably, this high level of accuracy persists across ONMI, Omega Index, and Soft Modularity, even in networks with severe noise and intense overlap. In addition to high accuracy, DPC-HOPE is computationally efficient ($O(m + n \log n)$). By employing a deterministic, interpretable propagation of community membership, it replaces stochastic heuristics with a principled framework, providing clear visibility into the topological structure of real-world data. In an era of increasingly complex and overlapping systems, DPC-HOPE emerges as a robust and scalable framework for fuzzy community detection, offering a principled approach that can drive insights across social, biological, and technological systems.

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