Adaptive Fuzzy Event-Triggered Quantized Control for Uncertain Nonlinear Systems

1st Qingtan Meng
School of Automation,
Nanjing University of Science and Technology
Jiangsu, China
qtmeng0609@163.com

2rd Qian Ma
School of Automation,
Nanjing University of Science and Technology
Jiangsu, China
qma@njust.edu.cn

***Abstract*—This paper addresses the adaptive control problem for a family of uncertain nonlinear systems. By employing fuzzy logic systems to approximate the unknown nonlinear terms of the systems, we develop an adaptive control scheme that incorporates a novel dynamic event-triggered quantized mechanism. The approach can effectively conserve communication resources without relying on input-to-state stability properties or assumptions. Theoretical analysis proves that all signals in the closed-loop systems are globally bounded and the Zeno phenomenon is excluded. Simulation results are provided to validate the effectiveness of the proposed control scheme.**

***Index Terms*—Fuzzy logic systems, event-triggered control, quantized control, nonlinear systems, uncertainties.**

I. Introduction

Networked control systems have garnered considerable research interest due to their broad applications in industrial automation [1], wireless sensor networks [2], smart grids [3] and other areas. A key feature of such systems is their reliance on shared communication networks for real-time data exchange among sensors, actuators and controllers. This architecture offers distinct advantages, including low cost, high flexibility, and ease of scalability and maintenance [4], [5]. However, the bandwidth of actual communication channels, particularly in wireless settings, is inherently limited. Therefore, investigating strategies for the efficient utilization of limited network resources to fulfill control tasks is of critical importance.

Event-triggered control has attracted much attention in recent years due to its capabilities in reducing data transmission and saving communication resources [6]. In contrast to the periodic nature of time-triggered control, it executes sampling and communication only when specific events occur. This action-on-demand mechanism eliminates redundant data transmissions, thus reducing network traffic and conserving bandwidth. On this basis, many representative works have been published successively on event-triggered control; see e.g., [7]–[11] and the references therein.

As modern communication channels are fundamentally digital, continuous-valued signals must be quantized into bit sequences for transmission [12]–[16]. This amplitude discretization reduces the size of individual data packets. It has been shown that a combined event-triggered and quantized control strategy can synergistically conserve communication resources by simultaneously reducing transmission frequency and limiting data accuracy, outperforming the use of either method alone. Some remarkable works include [17]–[20] for linear systems.

Nonlinearities always exist in many practical systems, thus recent attention has been directed to event-triggered quantized control problem for nonlinear systems [21]–[25]. However, it should be pointed out that the event-triggering and quantization mechanisms of the above literature are static. Given the greater advantage of dynamic triggering and quantization in terms of efficient resource utilization, the authors in [26] concerned the dynamic event-based quantized control for nonlinear systems via small-gain method. In [27] and [28], an output feedback controller was designed for nonlinear systems with dynamic state event-triggering and quantization, respectively. In [29], an asynchronous dynamic quantizer-dependent event-triggered control scheme was proposed.

Notably, full knowledge of the system models in the above literature is required, which leads to the proposed control strategies lacking the adaptability to deal with uncertainties commonly encountered in real-world applications. Moreover, existing works on dynamic event-triggered quantized control methods all relies on the input-to-state stability (ISS) condition, which is generally quite restrictive and difficult to verify. This motivates us to raise the following question: *Can we propose an adaptive control scheme that does not depend on the ISS assumption for uncertain nonlinear systems subject to dynamic event-triggering and quantization mechanism?* In this paper, we are committed to addressing this challenge. The main contributions of this work are summarized below.

1) Compared with [26]–[29] where full knowledge of the system nonlinearities is required, more general system models containing completely unknown nonlinear functions are considered. Under this configuration, dynamic event-triggered quantization mechanism is introduced, which is more in line with practical digital calculations and significantly reduces resource burdens.

2) A novel adaptive event-triggered quantized control

This work was supported in part by the NSFC under Grant 62473199, 62503228, 62533013, the Postdoctoral Fellowship Program of CPSF under Grant GZB20250958, and Jiangsu Funding Program for Excellent Postdoctoral Talent. *(Corresponding author: Qian Ma)*

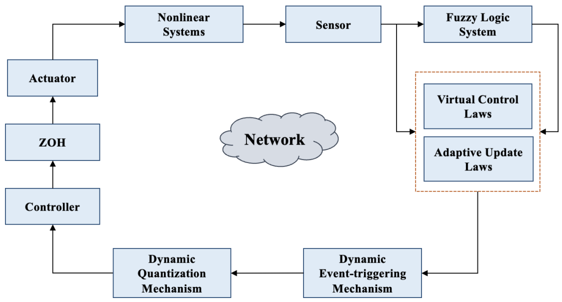

Fig. 1. Control block diagram.

scheme is developed by developing fuzzy logic systems to approximate unknown nonlinear terms of systems. More importantly, the ISS assumption necessary for [26]–[29] is removed successfully. It is guaranteed that all the signals of the closed-loop systems are globally bounded and the Zeno behavior can be excluded.

Notations: For a given matrix P, $\|P\|$ is its Euclidean norm and P^T is its transpose. $\mathbb{R}^i$ denotes Euclidean space with dimension i.

II. Preliminaries and Problem Formulation

In this paper, the system model is described as follows:

$$\begin{aligned} \dot{x}_i &= x_{i+1} + f_i(\bar{x}_i), \quad i = 1, \cdots, n-1, \\ \dot{x}_n &= u + f_n(x), \end{aligned} \tag{1}$$

where $x = [x_1, \cdots, x_n]^T \in \mathbb{R}^n$ is the system state, $\bar{x}_i = [x_1, \cdots, x_i]^T$, $u \in \mathbb{R}$ is the control input, $f_i(\bar{x}_i) : \mathbb{R}^i \to \mathbb{R}$ is the unknown smooth nonlinear function with $f_i(0) = 0$.

In this paper, we use fuzzy logic systems (FLSs) to approximate unknown functions on some compact set. The FLSs are described as follows.

If-Then Rules:

Rule$^{(l)}$*: If* z_1 is A_1^l, z_2 is A_2^l, $\cdots$, z_k is A_k^l

Then: y_B is B^l

where $z = [z_1, \cdots, z_k]^T \in \mathbb{R}^k$ is the input variable of FLSs and y_B is the output variable. l defines the number of fuzzy rules. $\mu_{A_i^l}(z_i)$ and $\mu_{B^l}(y_B)$ are the membership functions of the fuzzy sets A_i^l and B^l.

Based on the center average defuzzifier, product inference, Singleton fuzzifier, FLSs is expressed as

$$y_B(z) = \frac{\sum_{l=1}^{L} \Theta_l \prod_{i=1}^{k} \mu_{A_i^l}(z_i)}{\sum_{l=1}^{L} [\prod_{i=1}^{k} \mu_{A_i^l}(z_i)]} \tag{2}$$

where Θ_l satisfies $\mu_{B^l}(\Theta_l) = \max\{\mu_{B^l}(y_B) | y_B \in \mathbb{R}\}$.

Let

$$\Theta_l = \frac{\prod_{i=1}^{k} \mu_{A_i^l}(z_i)}{\sum_{l=1}^{L} [\prod_{i=1}^{k} \mu_{A_i^l}(z_i)]}, \; l = 1, \cdots, L \tag{3}$$

where $\theta = [\theta_1, \cdots, \theta_L]^T$ is a fuzzy weighting vector, $\Theta(x) = [\Theta_1, \cdots, \Theta_L]^T$ is a basis function vector. Then, (2) is rewritten as

$$y_B(z) = \theta^T \Theta(z). \tag{4}$$

Lemma 1 [30]: For a continuous function $f(x)$ defined on a compact set Ω, there exist a FLS (4) satisfying

$$\sup_{x \in \Omega} |f(z) - \theta^T \Theta(z)| \leq \epsilon \tag{5}$$

where $\epsilon > 0$ is a constant.

III. Controller Design

In this section, we provide the procedure of adaptive fuzzy event-triggered quantized controller design via the backstepping method. The control block is shown in Fig. 1.

First, the coordinate transformation is introduced as

$$\zeta_1 = x_1, \quad \zeta_i = x_i - \alpha_i(\bar{x}_{i-1}, \bar{\hat{\theta}}_{i-1}) \tag{6}$$

where $i = 2, \cdots, n$; $\bar{\hat{\theta}}_{i-1} = [\hat{\theta}_1, \hat{\theta}_2, \cdots, \hat{\theta}_{i-1}]^T$ with $\hat{\theta}_j$ being the estimate of the unknown constant $\theta_j^* = \|\theta_j\|^2$; α_i is the virtual control law to be designed.

Now, we elaborate the design procedure. It follows from (1) and (6) that

$$\begin{aligned} \dot{\zeta}_1 &= \zeta_2 + \alpha_2 + f_1, \\ \dot{\zeta}_i &= \zeta_{i+1} + \alpha_{i+1} + f_i + \Pi_i, \\ \dot{\zeta}_n &= u + f_n + \Pi_n, \end{aligned} \tag{7}$$

where $i = 2, \cdots, n-1$ and $\Pi_i = -\sum_{j=1}^{i-1} \frac{\partial \alpha_i}{\partial x_j}(x_{j+1} + f_j) - \sum_{j=1}^{i-1} \frac{\partial \alpha_i}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j$. With system (7), we construct the Lyapunov function as follows

$$V_\zeta = \sum_{i=1}^{n} (\frac{1}{2}\zeta_i^2 + \frac{1}{2\Gamma_i}\tilde{\theta}_i^2), \tag{8}$$

where $\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i$. Then, the derivative of V_ζ can be calculated as

$$\begin{aligned} \dot{V}_\zeta =& \sum_{i=1}^{n-1} \zeta_i \zeta_{i+1} + \sum_{i=1}^{n-1} \zeta_i \alpha_{i+1} + \zeta_n u + \sum_{i=1}^{n} \zeta_i f_i \\ &+ \sum_{i=2}^{n} \zeta_i \Pi_i - \sum_{i=1}^{n} \frac{1}{\Gamma_i} \tilde{\theta}_i \dot{\hat{\theta}}_i \\ =& \sum_{i=1}^{n-1} \zeta_i (\alpha_{i+1} + g_i(\xi_i)) - \sum_{i=1}^{n} \frac{1}{\Gamma_i} \tilde{\theta}_i \dot{\hat{\theta}}_i \\ &+ \zeta_n (u + g_n(\xi_n)), \end{aligned} \tag{9}$$

where $g_1(\xi_1) = f_1$ and $g_i(\xi_i) = \zeta_{i-1} + f_i + \Pi_i$ with $\xi_i = [\bar{x}_i^T, \bar{\hat{\theta}}_{i-1}^T]^T$ for $i = 2, \cdots, r$.

Since the nonlinear function f_i in (1) is unknown, this makes g_i also unknown. Therefore, by Lemma 2, there exist FLSs $\theta_i^T \Theta_i(\xi_i)$ such that

$$g_i(\xi_i) = \theta_i^T \Theta_i(\xi_i) + \epsilon_i(\xi_i), \; |\epsilon_i(\xi_i)| \leq \bar{\epsilon}_i, i = 1, \cdots, r,$$

where $\bar{\epsilon}_i > 0$ are known constants. Then, it is easy to calculate that

$$\begin{aligned}\zeta_i g_i(\xi_i) =&\zeta_i(\theta_i^T \Theta_i(\xi_i) + \epsilon(\xi_i))\\ \leq&\frac{\theta_i^*}{2\varepsilon_i^2}\Theta_i(\xi_i)\Theta_i^T(\xi_i)\zeta_i^2 + \frac{1}{2}\varepsilon_i^2 + \frac{1}{2}\zeta_i^2 + \frac{1}{2}\bar{\epsilon}_i^2, \quad (10)\end{aligned}$$

where $\varepsilon_i > 0$ are design parameters. Substituting (10) into (9), we have

$$\begin{aligned}\dot{V}_\zeta \leq& \sum_{i=1}^{n-1} \zeta_i(\alpha_{i+1} + \frac{1}{2}\zeta_i + \frac{1}{2\varepsilon_i^2}\hat{\theta}_i\zeta_i) + \sum_{i=1}^{n}\frac{1}{2}\varepsilon_i^2\\ &+ \zeta_n(u + \frac{1}{2}\zeta_n + \frac{1}{2\varepsilon_n^2}\hat{\theta}_n\zeta_n) + \sum_{i=1}^{n}\frac{1}{2}\bar{\epsilon}_i^2\\ &- \sum_{i=1}^{n}\frac{1}{\Gamma_i}\tilde{\theta}_i(\hat{\theta}_i - \frac{\Gamma_i}{2\varepsilon_i^2}\zeta_i^2). \quad (11)\end{aligned}$$

Design the virtual control law α_{i+1} for $i = 1, \cdots, n$ as

$$\alpha_{i+1} = -b_i\zeta_i - \frac{1}{2}\zeta_i - \frac{\hat{\theta}_i}{2\varepsilon_i^2}\Theta_i(\xi_i)\Theta_i^T(\xi_i)\zeta_i, \quad (12)$$

where $b_i > 0$ are design parameters. Then, $\hat{\theta}_i$ is updated by

$$\dot{\hat{\theta}}_i = -\delta_i\hat{\theta}_i + \frac{\Gamma_i}{2\varepsilon_i^2}\zeta_i^2 \quad (13)$$

where $\Gamma_i > 0$ and $\delta_i > 0$ are design parameters. On this basis, the inequality (11) can be rewritten as

$$\begin{aligned}\dot{V}_\zeta \quad \leq& \quad -\sum_{i=1}^{n} b_i\zeta_i^2 + \zeta_n(u - \alpha_{n+1})\\ &+ \sum_{i=1}^{n}(\frac{1}{2}\varepsilon_i^2 + \frac{1}{2}\bar{\epsilon}_i^2 + \frac{\delta_i}{\Gamma_i}\tilde{\theta}_i\hat{\theta}_i). \quad (14)\end{aligned}$$

In conventional control schemes, the controller is typically defined as $u = \alpha_{n+1}$. This implies that the control action is continuously applied to the system, regardless of its actual need adjustment, resulting in unnecessary resource expenditure. To address this issue, a dynamic event-triggered quantized mechanism is employed in this work. The triggering condition for α_{n+1} is defined as

$$\begin{cases} \alpha_{n+1}^s(t) = \alpha_{n+1}(t_l), \quad \forall t \in [t_l, t_{l+1}), \\ t_{l+1} = \inf\{t > t_l : |\tilde{\alpha}(t)| \geq \frac{\rho_1}{r(t)}\}, \end{cases} \quad (15)$$

where $\tilde{\alpha}(t) = \alpha_{n+1}^s(t) - \alpha_{n+1}(t)$, $\rho_1 > 0$ is a design parameter, t_l ($l \in \mathbb{Z}^+$) is the update time. During the time $t \in [t_l, t_{l+1})$, the control signal holds as the constant $\alpha_{n+1}(t_l)$. $r(t)$ is a dynamic parameter generated by

$$\dot{r}(t) = \alpha\varphi(M - r(t)), \quad r(0) = 1, \quad (16)$$

where $\alpha > 0$ and $M > 1$ are design parameters, $\varphi(M - r(t))$ is a continuous design function where $\varphi(M - r(t)) > 0$ with $r(t) < M$ and $\varphi(0) = 0$.

Then, the triggered control signal $\alpha_{n+1}^s(t)$ is quantized at the encoder side. The quantizer is modeled as follows

$$\alpha_{n+1}^{sq}(t) = q(\alpha_{n+1}^s(t)) = q(\alpha_{n+1}(t_l)), \quad \forall t \in [t_l, t_{l+1}), \quad (17)$$

which satisfies the following property

$$|q(\alpha_{n+1}^s(t)) - \alpha_{n+1}^s(t)| \leq \frac{\rho_2}{r(t)}, \quad (18)$$

where $\rho_2 > 0$ is a design parameter.

Let the actual controller u as

$$u(t) = \alpha_{n+1}^{sq}(t), \quad \forall t \in [t_l, t_{l+1}). \quad (19)$$

It follows from the dynamic event-triggering mechanism and quantization mechanism that

$$\begin{aligned}\zeta_n(u - \alpha_{n+1}) \quad =& \quad \zeta_n(\alpha_{n+1}^{sq} - \alpha_{n+1}^s + \alpha_{n+1}^s - \alpha_{n+1})\\ \leq& \quad |\zeta_n|(|\alpha_{n+1}^{sq} - \alpha_{n+1}^s| + |\alpha_{n+1}^s - \alpha_{n+1}|)\\ \leq& \quad |\zeta_n|(\frac{\rho_1 + \rho_2}{r})\\ \leq& \quad \frac{\beta_1}{2}\zeta_n^2 + \frac{1}{2\beta_1}(\rho_1 + \rho_2)^2, \quad (20)\end{aligned}$$

where $\beta_1 > 0$ is a design parameter.

Then, (14) can be rewritten as

$$\begin{aligned}\dot{V}_\zeta \quad \leq& \quad -\sum_{i=1}^{n-1} b_i\zeta_i^2 - (b_n - \frac{\beta_1}{2})\zeta_n^2 + \frac{1}{2\beta_1}(\rho_1 + \rho_2)^2\\ &+ \sum_{i=1}^{n}(\frac{1}{2}\varepsilon_i^2 + \frac{1}{2}\bar{\epsilon}_i^2 + \frac{\delta_i}{\Gamma_i}\tilde{\theta}_i\hat{\theta}_i). \quad (21)\end{aligned}$$

Remark 1: In networked control systems, the communication channels have the requirement of saving bandwidth. Embedding event-triggers and quantizers simultaneously can suppress the issuance of redundant control instructions, while compress the transmitted input information. In this way, the efficiency of data transmission in communication can be improve effectively. This operation also aligns well with the practical digital signal processing chain, where analog signals are invariably first sampled and then quantized for digital conversion.

Remark 2: It can be readily deduced that the function $r(t)$ monotonically increases from 1 to M. Through qualitative analysis, this characteristic has the advantage in effectively balancing the relationship between communication resources and control performance. Specifically, when the system state or control signal exhibits large magnitudes, the trigger threshold and quantization accuracy can be relaxed to tolerate larger measurement errors to reduce unnecessary sampling. As the system state or control signal approaches the origin, a smaller trigger threshold and a higher quantization accuracy can make the system control more precise and thus enhance the system performance. More crucially, $r(t)$ is a dynamically generated parameter rather than a static calculation based on the last transmitted value. This affords superior flexibility, allowing the trigger threshold and quantization accuracy to adapt in accordance with the real-time evolution of the system.

Remark 3: In this paper, a dynamic parameter $r(t)$ is introduced within the adaptive fuzzy control scheme. The parameter $r(t)$ is designed as a system signals-independent function, which embedded in both event-triggering mechanism and quantization mechanism to avoid the waste of communication resources in a flexible manner. All design parameters in the dynamic equation of $r(t)$ are subject solely to the lenient conditions given in (16). Observe that the design function $\phi(M-r(t))$ is not defined explicitly, so the form of (16) is highly flexible and encompasses numerous possible implementations. In particular, the first-order nonlinear differential equation in [8] is recovered as a special case of our general framework.

IV. Main Results

Now, we state the stability analysis.

Theorem 1: Consider the uncertain nonlinear system described by (1) in the presence of dynamic event triggering and quantization mechanisms (15)-(18). By applying the adaptive fuzzy controller (19) with (12) and (13), all the signals of the closed-loop system are globally bounded. Furthermore, the occurrence of the Zeno phenomenon is eliminated.

Proof: In (21), it follows from Young's inequality that

$$\frac{\delta_i}{\Gamma_i}\tilde{\theta}_i\hat{\theta}_i \leq -\frac{\delta_i}{2\Gamma_i}\tilde{\theta}_i^2 + \frac{\delta_i}{2\Gamma_i}(\theta_i^*)^2. \tag{22}$$

Combining (21) with (22), one obtains that

$$\begin{aligned} \dot{V}_\zeta \leq & -\sum_{i=1}^{n-1} b_i\zeta_i^2 - (b_n - \frac{\beta_1}{2})\zeta_n^2 - \sum_{i=1}^{n}\frac{\delta_i}{2\Gamma_i}\tilde{\theta}_i^2 \\ & + \sum_{i=1}^{n}(\frac{1}{2}\varepsilon_i^2 + \frac{1}{2}\bar{\epsilon}_i^2 + \frac{\delta_i}{2\Gamma_i}(\theta_i^*)^2) \\ & + \frac{1}{2\beta_1}(\rho_1+\rho_2)^2. \end{aligned} \tag{23}$$

By choosing $\bar{c}_i = b_i$ for $i = 1, \cdots, n-1$, $\bar{c}_n = b_n - \frac{\beta_1}{2} > 0$, and defining $\Theta = \sum_{i=1}^{n}(\frac{1}{2}\varepsilon_i^2 + \frac{1}{2}\bar{\epsilon}_i^2 + \frac{\delta_i}{2\Gamma_i}(\theta_i^*)^2) + \frac{1}{2\beta_1}(\rho_1+\rho_2)^2$, the derivative of V_ζ is deduced as

$$\dot{V}_\zeta \leq -\sum_{i=1}^{n}\bar{c}_i\zeta_i^2 - \sum_{i=1}^{n}\frac{\delta_i}{2\Gamma_i}\tilde{\theta}_i^2 + \Theta \leq -\bar{c}V_\zeta + \Theta, \tag{24}$$

where $\bar{c} = \min\{2\bar{c}_1, \cdots, 2\bar{c}_n, \delta_1, \cdots, \delta_n\}$ is a positive design parameter. Multiplying (24) by $e^{\bar{c}t}$ and integrating it on $[0, t]$, one gets

$$V_\zeta \leq V_\zeta(0)e^{-\bar{c}t} + \frac{\Theta}{\bar{c}}. \tag{25}$$

Hence, there exists a time $T = \max\{0, \frac{1}{\bar{c}}\ln(\frac{\bar{c}V(0)}{\sigma\Theta})\}$ for any $0 < \sigma < 1$ satisfying $V \leq (1+\sigma)\frac{\Theta}{\bar{c}}$, $\forall t \geq T$. From the construction of V_ζ, one further has

$$\zeta_i^2 \leq 2V_\zeta \leq 2(1+\sigma)\frac{\Theta}{\bar{c}} \tag{26}$$

$$\tilde{\theta}_i^2 \leq 2\Gamma_i V \leq 2\Gamma_i(1+\sigma)\frac{\Theta}{\bar{c}} \tag{27}$$

which means that ζ_i and $\tilde{\theta}_i$ are bounded. Thus, x_i and α_i is bounded. It follows from $\tilde{\theta}_i = \theta_i^* - \hat{\theta}_i$ that $\hat{\theta}_i$ is bounded. Based on (15)-(19), the control input u is bounded. In conclusion, all signals of the closed-loop systems are bounded.

Now, we perform the analysis to exclude the Zeno phenomenon. Recall that $\tilde{\alpha}(t) = \alpha_{n+1}^s(t) - \alpha_{n+1}(t)$, $\forall t \in [t_l, t_{l+1})$. The derivative of $\tilde{\alpha}(t)$ is calculated as $\frac{d}{dt}\|\tilde{\alpha}(t)\| = \frac{d}{dt}(\tilde{\alpha}^T(t) * \tilde{\alpha}(t))^{\frac{1}{2}} = sign(\tilde{\alpha}(t))\dot{\tilde{\alpha}}(t) \leq \|\dot{\alpha}_{n+1}(t)\|$. By (12), $\dot{\alpha}_{n+1}$ must be a continuous function. It follows from the above stability analysis that all the variables of $\dot{\alpha}_{n+1}$ are bounded. Hence, there exists a positive constant ν_1 that $\|\dot{\alpha}_{n+1}(t)\| \leq \nu_1$. According to $\tilde{\alpha}(t_l) = 0$ and $\lim_{t\to t_{l+1}}\tilde{\alpha}(t) = \frac{\rho_1}{r(t)} \geq \frac{\rho_1}{M}$, the lower bound of inter-execution intervals of the state event trigger must satisfy $T_1 \geq \frac{\rho_1}{M\nu_1}$. Therefore, the Zeno behavior can be avoided for the event-trigger (15). In this way, the proof of Theorem 1 is complete. ■

V. Simulation

Consider the following system adapted from Chua's circuit

$$\begin{aligned} \dot{V}_{C_1} &= \frac{1}{C_1R}(V_{C_2} - V_{C_1}) - \frac{1}{C_1}f(V_{C_1}), \\ \dot{V}_{C_2} &= \frac{1}{C_2R_1}(V_{C_2} - V_{C_1}) - \frac{1}{C_2}I_L, \\ \dot{I}_L &= \frac{1}{L}u - \frac{1}{LR_2}V_{C_2} - \frac{R_0}{LR_2}I_L, \end{aligned} \tag{28}$$

where V_{C_1} and V_{C_2} represent the voltages across the capacitors C_1 and C_2, respectively; I_L represents the current through the inductor L, u is the actual control input; $f(V_{C_1})$ is the current through Chua's diode, which is a nonlinear function. For simulation, define $C_1 = C_2 = 1$, $R = 1$, $R_0 = 1$, $R_1 = 10$, $R_2 = 2$, $L = 1$, and $f(V_{C_1}) = -V_{C_1} + V_{C_1}\sin(V_{C_2})$.

Let $x_1 = V_{C_1}$, $x_2 = V_{C_2}$ and $x_3 = I_L$, system (28) can be rewritten as follows

$$\begin{aligned} \dot{x}_1 &= x_2 - x_1\sin(x_2), \\ \dot{x}_2 &= x_3 + \frac{1}{10}x_2 - \frac{1}{10}x_1, \\ \dot{x}_3 &= u - \frac{1}{2}x_2 - \frac{1}{2}x_3. \end{aligned} \tag{29}$$

Following the backstepping design procedure in Section III, we can develop the adaptive fuzzy event-triggered quantized controller as

$$\left\{ \begin{aligned} \zeta_1 &= x_1, \\ \zeta_2 &= x_2 - \alpha_2, \\ \zeta_3 &= x_3 - \alpha_3, \\ \alpha_2 &= -b_1\zeta_1 - \tfrac{1}{2}\zeta_1 - \tfrac{1}{2\varepsilon_1^2}\hat{\theta}_1\zeta_1, \\ \alpha_3 &= -b_2\zeta_2 - \tfrac{1}{2}\zeta_2 - \tfrac{1}{2\varepsilon_2^2}\hat{\theta}_2\zeta_2, \\ \alpha_4 &= -b_3\zeta_3 - \tfrac{1}{2}\zeta_3 - \tfrac{1}{2\varepsilon_3^2}\hat{\theta}_3\zeta_3, \\ \dot{\hat{\theta}}_1 &= -\delta_1\hat{\theta}_1 + \tfrac{\Gamma_1}{2\varepsilon_1^2}\zeta_1^2, \\ \dot{\hat{\theta}}_2 &= -\delta_2\hat{\theta}_2 + \tfrac{\Gamma_2}{2\varepsilon_2^2}\zeta_2^2, \\ \dot{\hat{\theta}}_3 &= -\delta_2\hat{\theta}_2 + \tfrac{\Gamma_3}{2\varepsilon_2^2}\zeta_3^2, \\ \dot{r} &= -\tfrac{1}{2}r^2 + 3r, r(0) = 1, \\ u &= \alpha_4^{sq}, \end{aligned} \right. \tag{30}$$

where the parameter ρ_1 in the triggering threshold is chosen as $\rho_1 = 1$, and the dynamic quantizer is designed as

$$q(\mathfrak{z}) = \left\{ \begin{array}{ll} \mathfrak{z}_i\mathrm{sgn}(\mathfrak{z}), & \mathfrak{z}_i - \frac{r}{2} \leq \mathfrak{z} \leq \mathfrak{z}_i - \frac{r}{2}, \\ 0, & |\mathfrak{z}| \leq \mathfrak{z}_0, \end{array} \right. \tag{31}$$

where $\mathfrak{z}_0 = \varepsilon_{i+1}\eta$ for $i = 3, 4$, $\mathfrak{z}_1 = \mathfrak{z}_0 + \frac{r}{2}$, $\mathfrak{z}_j = \mathfrak{z}_{j-1} + r$ with $j = 2, \cdots$, and $r = 2\varepsilon_{i+1}\eta$ is the length of the quantization interval. Clearly, the property (18) is satisfied for (31).

In simulation, choose the design parameters as $b_1 = 3$, $b_2 = 4$, $b_3 = 3$, $\delta_1 = \delta_2 = \delta_3 = 1.5$, $\Gamma_1 = 0.55$, $\Gamma_2 = 0.1$, $\Gamma_3 = 2.05$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.707$, $\rho_2 = 1$. The fuzzy membership functions are selected as $\mu_{f_i^l} = \exp[-\frac{(z_i-2+0.5l)^2}{8}]$ with $l = 1, \cdots, 7$. The initial conditions are chosen as $x_1(0) = 3$, $x_2(0) = 0$, $x_3(0) = -3$, $\hat{\theta}_1(0) = \hat{\theta}_2(0) = \hat{\theta}_3(0) = 0$. The simulation step size is set to $0.0001s$. Simulation results in Figs. 2-4 indicate that all the signals of the closed-loop system converge to a neighborhood of the origin within approximately $6s$ and remain in that vicinity thereafter. Under the given step size, a time-triggered controller would require $5 * 10^6$ transmissions during the initial $500s$. Under the event-triggered controller proposed in this paper, the number of recorded transmissions across the communication channel is 14248. This corresponds to a 99.71% reduction in the total number of transmissions. These results clearly highlight the advantage of the proposed dynamic event-triggered control algorithms in the sense of reducing communication traffic.

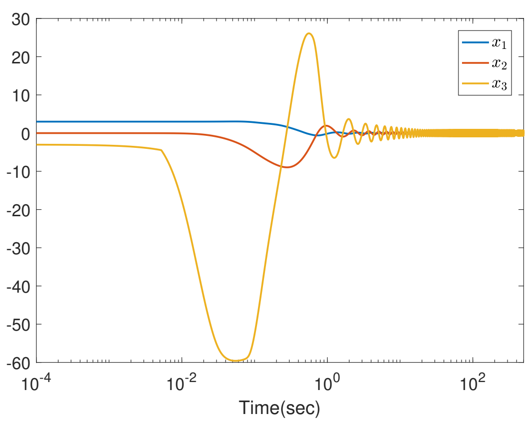

Fig. 2 Curves of x_1, x_2 and x_3

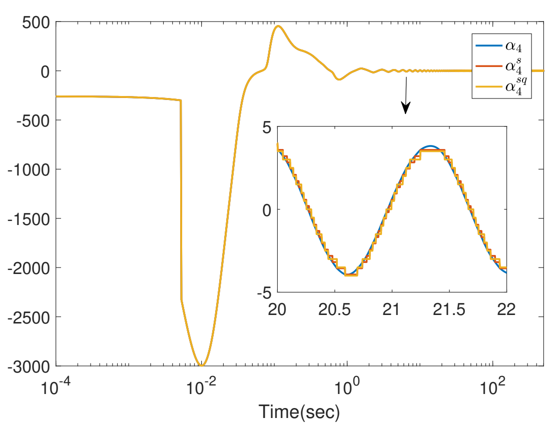

Fig. 3 Curves of α_4, α_4^s and α_4^{sq}

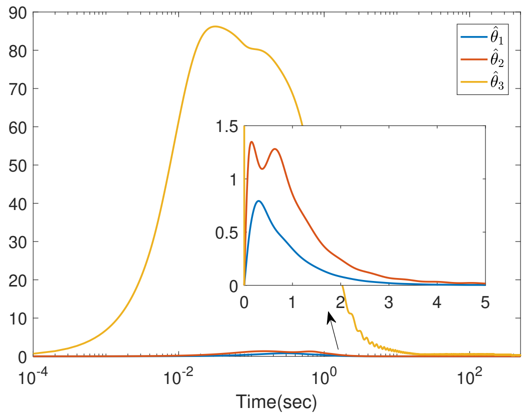

Fig. 4 Curves of $\hat{\theta}_1$, $\hat{\theta}_2$ and $\hat{\theta}_3$

VI. Conclusion

In this paper, we investigate the control problem for uncertain nonlinear systems, in which fuzzy logic systems are employed to approximate the unknown nonlinearities. Considering that the resources in the communication channel are limited, a dynamic event-triggered quantized mechanism is constructed. Without the ISS assumptions, an adaptive fuzzy event-triggered quantized control scheme is constructed by developing the backstepping design procedure. According to Lyapunov analysis method, it is proven that all closed-loop signals are globally bounded and the Zeno behavior is excluded. As a promising direction for future research, it would be interesting to extend the control framework to achieve faster stability properties of the nonlinear systems, such as finite-time or fixed-time stability.

References

[1] J. García, F. Palomo, A. Luque, et al. "Reconfigurable distributed network control system for industrial plant automation," *IEEE Transactions on Industrial Electronics*, 2004, 51(6): 1168-1180.

[2] Y. Wang, S. Wu, Y. Wang, et al. "Goal-oriented transmission scheduling for energy-efficient wireless networked control in SAGIN: An AoI-thresholding mechanism", *IEEE Transactions on Vehicular Technology*, 2024, 73(8): 11503-11517.

[3] A. Singh, R. Singh, B. Pal. "Stability analysis of networked control in smart grids," *IEEE Transactions on Smart Grid*, 2014, 6(1): 381-390.

[4] J. P. Hespanha, P. Naghshtabrizi, Y. Xu. "A survey of recent results in networked control systems," *Proceedings of the IEEE*, 2007, 95(1): 138–162.

[5] X. Zhang, Q. Han, X. Ge, et al. "Networked control systems: A survey of trends and techniques," *IEEE/CAA Journal of Automatica Sinica*, 2019, 7(1): 1-17.

[6] W. Heemels, K. Johansson, P. Tabuada. "An introduction to event-triggered and self-triggered control," *2012 IEEE Conference on Decision and Control*, 2012: 3270-3285.

[7] L. Xing, C. Wen. "Dynamic event-triggered adaptive control for a class of uncertain nonlinear systems," *Automatica*, 2023, 158: 111286.

[8] H. Ye, Y. Song, Z. Zhang, et al. "Global dynamic event-triggered control for nonlinear systems with sensor and actuator faults: A matrix-pencil-based approach," *IEEE Transactions on Automatic Control*, 2023, 69(3): 2007-2014.

[9] X. Zhang, Q. Han, B. Zhang. "An overview and deep investigation on sampled-data-based event-triggered control and filtering for networked systems," *IEEE Transactions on Industrial Informatics*, 2016, 13(1): 4-16.

[10] J. Huang, W. Wang, C. Wen, et al. "Adaptive event-triggered control of nonlinear systems with controller and parameter estimator triggering," *IEEE Transactions on Automatic Control*, 2019, 65(1): 318-324.

[11] T. Liu, Z. Jiang. "A small-gain approach to robust event-triggered control of nonlinear systems," *IEEE Transactions on Automatic Control*, 2015, 60(8): 2072-2085.

[12] Z. Zhang, C. Wen, L. Xing, et al. "Adaptive output feedback control of nonlinear systems with mismatched uncertainties under input/output quantization," *IEEE Transactions on Automatic Control*, 2022, 67(9): 4801-4808.

[13] J. Zhou, C. Wen, W. Wang, et al. "Adaptive backstepping control of nonlinear uncertain systems with quantized states," *IEEE Transactions on Automatic Control*, 2019, 64(11): 4756-4763.

[14] Y. Zhou, A. Polyakov, G. Zheng, "Finite/fixed-Time stabilization of linear systems with state quantization," *IEEE Transactions on Automatic Control*, 2024, DOI: 10.1109/TAC.2024.3473619.

[15] Y. Li, G. Yang. "Adaptive asymptotic tracking control of uncertain nonlinear systems with input quantization and actuator faults," *Automatica*, 2016, 72: 177-185.

[16] H. Li, C. Hua, K. Li, et al, "Adaptive state-quantized control for mismatched nonlinear systems via a dynamic gain approach," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2023, DOI: 10.1109/TSMC.2023.3330991.

[17] M. Abdelrahim, V. Dolk, W. Heemels. "Event-triggered quantized control for input-to-state stabilization of linear systems with distributed output sensors," *IEEE Transactions on Automatic Control*, 2019, 64(12): 4952-4967.

[18] E. Garcia, P. Antsaklis. "Model-based event-triggered control for systems with quantization and time-varying network delays," *IEEE Transactions on Automatic Control*, 2012, 58(2): 422-434.

[19] Y. Shi, E. Nekouei. "Quantization and event-triggered policy design for encrypted networked control," *IEEE/CAA Journal of Automatica Sinica*, 2024, 11(4): 946-955.

[20] N. Lin, Q. Ling. "Event-triggered quantized consensus of linear multiagent systems under asynchronous denial-of-service attacks," *IEEE Transactions on Automatic Control*, 2024, 70(1): 697-704.

[21] H. Li, M. Li. "A dynamic gain method to event-triggered tracking control for uncertain nonlinear systems with input quantization," *IEEE Transactions on Automatic Control*, 2025, 70(6): 4093-4100.

[22] X. Cheng, K. Liu, Y. Wang, et al. "Safety-critical event-triggered control for networked control systems under quantization and time-varying delay," *IEEE Transactions on Control of Network Systems*, 2025, 12(3): 2219-2230.

[23] Z. Zhou, L. Zhao. "Event-triggered quantization-based predefined-time adaptive fuzzy control for quadrotor trajectory tracking, *IEEE Transactions on Artificial Intelligence*, 2026, 7(1): 596-605.

[24] Y. Jiang, Z. Liu, Z. Chen. "Fuzzy adaptive finite-time tracking control for a class of nonlinear systems: An event-triggered quantized control scheme," *IEEE Transactions on Fuzzy Systems*, 2023, 31(11): 4137-4144.

[25] T. Wang, H. Zhang, H. Zhang. "Fuzzy adaptive saturation control for nonlinear systems with event-triggered sampling and quantization," *IEEE Transactions on Automation Science and Engineering*, 2024, 22: 6954-6966.

[26] T. Liu, Z. Jiang. "Event-triggered control of nonlinear systems with state quantization," *IEEE Transactions on Automatic Control*, 2018, 64(2): 797-803.

[27] W. Liu, Q. Ma, S. Xu, et al. "State quantized output feedback control for nonlinear systems via event-triggered sampling," *IEEE Transactions on Automatic Control*, 2021, 67(12): 6810-6817.

[28] D. Almakhles, M. Abdelrahim. "Event-Triggered dynamic quantization for nonlinear systems with one-bit data transmission," *IEEE Transactions on Automatic Control*, 2023, 69(6): 4118-4125.

[29] T. Zhou, G. Yue, H. Li, et al. "Asynchronous quantizer-dependent event-triggered control for nonlinear networked control system with observer-based controller," *IEEE Transactions on Control of Network Systems*, 2023, 11(3): 1203-1213.

[30] Y. Xie, Q. Ma, J. Gu, and G. Zhou, "Event-triggered fixed-time practical tracking control for flexible-joint robot," *IEEE Trans. Fuzzy Syst.*, vol. 31, no. 1, pp. 67-76, Jan 2023.