PDC Output Feedback Control of T–S Fuzzy Systems with Premise Selection and Performance Constraints*

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***Abstract*—This paper addresses the PDC output feedback control design for nonlinear systems represented by a Takagi–Sugeno (T–S) fuzzy model. Sufficient conditions are derived in terms of linear matrix inequalities (LMIs) to guarantee that the closed-loop eigenvalues of the local models lie in a prescribed $\mathfrak{D}$-region in the complex plane, ensuring stability and desired transient performance. To handle the inherent non-convexity of the output feedback design, a two-stage matrix decomposition strategy is adopted to improve numerical feasibility. Moreover, beyond the matrix decomposition itself, a systematic selection of premise variables for the feedback controller is proposed, providing additional design flexibility and leading to less conservative results. A numerical example illustrates the effectiveness of the proposed approach.**

***Index Terms*—output feedback, Takagi-Sugeno (T-S) fuzzy models, $\mathfrak{D}$-region, parallel distributed compensation (PDC), selection of premise variables.**

I. INTRODUCTION

Takagi–Sugeno (T–S) fuzzy models provide an effective framework for representing nonlinear systems by combining local linear dynamics through **IF–THEN** rules [1], [2]. This structure enables the application of convex optimization techniques for analysis and control design.

$\mathfrak{D}$-region is a powerful approach for enforcing stability and dynamic performance by constraining the closed-loop eigenvalues to predefined regions of the complex plane [3]. Although originally proposed for linear systems, this concept has been successfully extended to T–S fuzzy models with local models subject to $\mathfrak{D}$-region constraints, enabling direct control over transient behavior [4], [5].

In the context of T–S fuzzy systems, the Parallel Distributed Compensation (PDC) controller is among the earliest nonlinear control strategies proposed in the literature [6], [7]. This approach consists of a fuzzy combination of local linear controllers, designed in parallel with the T–S model's rules.

Output feedback is particularly attractive in practical applications, despite the inherent non-convexity of its design [8], [9]. In [10], a matrix decomposition method used the system output matrix structure to reduce conservatism in LMI solutions. This efficient method was used in [5] to design output feedback in fuzzy T-S systems. Later, a new matrix decomposition was presented in [11]; the result introduces additional decision variables into the LMIs, yielding less conservative conditions. Finally, in [12], the authors proposed adding more decision variables to the matrix decomposition, at the cost of increasing the complexity of the control input. Due to this characteristic, the result was applied to the design of output feedback in fuzzy T-S systems, considering only a fixed gain.

The main contributions are i) novel LMI conditions for $\mathfrak{D}$-region constraint of T–S fuzzy systems via PDC output feedback, and ii) a systematic method for selecting premise variables to enable the implementation of fuzzy PDC controllers. The proposed result yields more general stability conditions than previous results in the literature [5], [11], [12], which were based on fixed-gain control.

The remainder of this paper is organized as follows. Section II presents the system description, including the Takagi–Sugeno fuzzy representation and the PDC control formulation. Section III introduces the preliminaries, detailing the adopted $\mathfrak{D}$-region and the quadratic Lyapunov function. Section IV presents the main results, including the adaptation of the matrix decomposition lemma—originally developed for robust control—to the PDC framework, the premise variable selection strategy, and the proposed output feedback theorem. Section V presents the controller design and simulation results, and Section VI concludes the paper.

Notation: $\mathbb{K}_{n_r} = \{1, 2, \dots, n_r\}$, $n_r \in \mathbb{N}$; $\mathbb{R}^{n_x}$ represents a vector with n_x real elements; "$(\cdot)^T$" indicates the transposition of a matrix or vector; The symbol "$\star$" generically denotes each symmetric element; I denotes an identity matrix of proper order; He(.) is the Hermitian operator such that $\mathrm{He}(A) = A + A^T$; $\otimes$ denotes the Kronecker product of the matrices; $C^\dagger$ represents a pseudo Moore–Penrose inverse of C. The simplex unit is represented by

$$\Lambda_{n_r} = \left\{\xi \in \mathbb{R}^{n_r} : \textstyle\sum_{i=1}^{n_r} \xi_i = 1,\ \xi_i \geq 0, i \in \mathbb{K}_{n_r}\right\}. \quad (1)$$

Data and Code Availability: The designed controller, code, and data obtained in this work are available in [13].

II. PROBLEM DESCRIPTION

Consider the nonlinear system given by:

$$\dot{x}(t) = g_1(\zeta(t))x(t) + g_2(\zeta(t))u(t), \quad y(t) = C_y x(t), \quad (2)$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state vector of the system, $u(t) \in \mathbb{R}^{n_u}$ is the input vector, and $y(t) \in \mathbb{R}^{n_y}$ is the measure output. The matrix $C_y \in \mathbb{R}^{n_y \times n_x}$ is known, and the nonlinear functions $g_1(\cdot) : \mathbb{R}^{n_\zeta} \to \mathbb{R}^{n_x \times n_x}$, $g_2(\cdot) : \mathbb{R}^{n_\zeta} \to \mathbb{R}^{n_x \times n_u}$,

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depend on the vector $\zeta(t) \in \mathbb{R}^{n_\zeta}$. The elements of $\zeta(t)$, denoted by $\zeta_p(t)$, $p \in \mathbb{K}_{n_\zeta}$, are functions of the state vector $x(t)$, uncertain parameters, or unknown variables of the system.

Consider that the nonlinear system described in (2), using the procedure given in [14], can be exactly written as a T-S fuzzy model with n_r rules of the form

$$\begin{aligned} &\underline{\text{Rule } i}: \textbf{IF } \zeta_1(t) \text{ is } \nu_{i1} \text{ and } \dots \text{ and } \zeta_p(t) \text{ is } \nu_{in_p}, \\ &\textbf{THEN } \dot{x}(t) = A_i x(t) + B_i u(t), \quad y(t) = C_y x(t). \end{aligned} \tag{3}$$

Let $\nu_{ip}(\zeta_p(t))$ be the membership degree of the variable $\zeta_p(t)$ to the fuzzy set ν_{ip}, based on [14], and the definitions given in (1) and (3), the dynamics (2) can be exactly represented by

$$\dot{x}(t) = \sum_{i=1}^{n_r} \xi_i(\zeta(t)) \left(A_i x(t) + B_i u(t)\right), y(t) = C_y x(t), \tag{4}$$

in which (A_i, B_i, C_y) are known matrices of appropriate dimensions, $\xi_i(\zeta(t))$ are the normalized membership functions given by

$$\xi_i(\zeta(t)) = \prod_{k=1}^{n_p} w_k^{\ell_{ik}}(\zeta(t)), \tag{5}$$

with $i \in \{1, \dots, n_r\}$, $\ell_{ik} \in \{0,\ 1\}$, $k \in \{1, \dots, p\}$,

$$w_k^0(\zeta(t)) = \frac{\overline{nl_k} - nl_k(\zeta(t))}{\overline{nl_k} - \underline{nl_k}}, w_k^1(\zeta(t)) = 1 - w_k^0(\zeta(t)) \tag{6}$$

and $nl_k(\zeta(t)) \in [\underline{nl_k}\ ,\ \overline{nl_k}]$ are the nonlinearities [2].

By (5) and (6), $\xi(\zeta(t)) \in \mathbb{R}^{n_r}$ satisfy the convex properties (1). Let the T-S fuzzy system (4) with the following PDC control law given by [6]:

$$u(t) = \sum_{i=1}^{n_r} \xi_i(\zeta(t)) K_i y(t), \tag{7}$$

with $K_i \in \mathbb{R}^{n_u \times n_y}$.

From the T-S fuzzy system (4) and PDC controller (7), the closed-loop dynamics are expressed as:

$$\dot{x}(t) = \big(A(\xi) + B(\xi) K(\xi) C_y\big) x(t), \tag{8}$$

with $[A(\xi) \quad B(\xi) \quad K(\xi)] = \sum_{i=1}^{n_r} \xi_i(\zeta(t))[A_i \quad B_i \quad K_i]$.

III. PRELIMINARY

A. LMIs based $\mathfrak{D}$-region

In this section, the objective is to propose relaxed quadratic LMI conditions for the $\mathfrak{D}$-region of system (8) using PDC controllers defined in (7). In this context, the following fundamental definition of $\mathfrak{D}$-region and an associated lemma will be useful.

Definition 1: [3] A subset $\mathfrak{D}$ of the complex plane is called a LMI region if it is defined by the matrices $\mathfrak{N} = \mathfrak{N}^T \in \mathbb{R}^{d \times d}$ and $\mathfrak{M} \in \mathbb{R}^{d \times d}$ such that:

$$\mathfrak{D} = \left\{\lambda \in \mathbb{C} : \mathfrak{N} + \lambda \mathfrak{M} + \bar{\lambda} \mathfrak{M}^T \prec 0\right\}, \tag{9}$$

where d is refereed to as the order of the LMI region.

In what follows, we focus on the LMI region described below and shown in [4, Fig. 1]. This region is selected for its relevance to stability analysis in the context under consideration. It comprises the left half-plane defined by $\Re(\lambda) < \beta$, a conic sector with apex at $(\gamma, 0)$ and inner angle $\pi/2 - \phi$, and a circle centered at $(q, 0)$ with radius r. This choice leads to the following matrices:

$$\mathfrak{N} = \begin{bmatrix} -2\beta & 0 & 0 & 0 & 0 \\ 0 & -2\gamma\cos\phi & 0 & 0 & 0 \\ 0 & 0 & -2\gamma\cos\phi & 0 & 0 \\ 0 & 0 & 0 & -r & -q \\ 0 & 0 & 0 & -q & -r \end{bmatrix}, \tag{10}$$

$$\mathfrak{M} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \cos\phi & \sin\phi & 0 & 0 \\ 0 & -\sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{11}$$

For more details and illustrative examples of how the matrices $\mathfrak{N}$ and $\mathfrak{M}$ are defined for different LMI regions, see [3]. The following definition states the fundamental conditions for $\mathfrak{D}$-region.

Definition 2: [15], [16] Given an LMI region defined by (9), a nonlinear system $\dot{x}(t) = f(x(t))x(t)$ is said to be $\mathfrak{D}$-stable if there exists a Lyapunov function $\mathcal{V}(x(t))$ satisfying $\frac{1}{2}\frac{\dot{\mathcal{V}}(x(t))}{\mathcal{V}(x(t))} \in \mathfrak{D}$, i.e.:

$$\mathfrak{N} \otimes \mathcal{V}(x(t)) + \mathfrak{M} \otimes \frac{1}{2}\dot{\mathcal{V}}(x(t)) + \mathfrak{M}^T \otimes \frac{1}{2}\dot{\mathcal{V}}(x(t)) \prec 0. \tag{12}$$

IV. MAIN RESULTS

In PDC controller-based control designs, the state feedback gain has a structure of the type

$$K_{xj} = Y_j F^{-1}, \tag{13}$$

with $K_{xj} \in \mathbb{R}^{n_u \times n_x}$, $Y_j \in \mathbb{R}^{n_u \times n_x}$, $F \in \mathbb{R}^{n_x \times n_x}$, and the matrices Y_j and F are obtained from the numerical solution of the LMIs. Since the objective of this work is to obtain a gain matrix for the output feedback, then the equality (13) is changed to

$$K_j C_y = Y_j F^{-1}, \tag{14}$$

where $K_j \in \mathbb{R}^{n_u \times n_y}$ is the gain used in the T-S fuzzy system (8). The presence of the matrix C_y in (14) introduces an additional challenge in obtaining the gain K_j.

Lemma 1: Given matrices $L \in \mathbb{R}^{n_x - n_y \times n_y}$ and $Q \in \mathbb{R}^{n_x \times n_x - n_y}$, such that $C_y Q = 0$. Let

$$\begin{cases} R = {C_y}^\dagger + QL, \\ F = \begin{bmatrix} Q & R \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} \\ 0 & F_{22} \end{bmatrix} \begin{bmatrix} Q^T \\ R^T \end{bmatrix}, \\ Y_j = Y_{Rj} R^T, \end{cases} \tag{15}$$

with $Y_{Rj} \in \mathbb{R}^{n_u \times n_y}$, $F_{11} \in \mathbb{R}^{n_x - n_y \times n_x - n_y}$, $F_{12} \in \mathbb{R}^{n_x - n_y \times n_y}$, $F_{22} \in \mathbb{R}^{n_y \times n_y}$, and ${C_y}^\dagger = C_y^T (C_y C_y^T)^{-1}$. If F is nonsingular, then F_{22} is also nonsingular and a solution to (14) is given by $K_j = Y_{Rj} F_{22}^{-1}$.

Proof: Replacing the fixed gain $K = Y_R F_{22}$ for the local gains $K_j = Y_{Rj} F_{22}^{-1}$, the proof follows the same steps from [11, Theorem 1].

This work extends Lemma 1, which exploits the structure of C_y to enable output feedback design for uncertain linear systems, to the case of nonlinear systems represented by T–S fuzzy models via a PDC controller. The key challenge is selecting the premise variables to construct the PDC control.

A. Premise variable selection

In the design of output feedback controllers, not all state variables are available in real time. Consequently, some premise variables may also be unavailable for feedback. In this section, it is assumed that at least one premise variable is measurable in real time. The main idea is to exploit information from all available premise variables to construct the PDC controller. From (5), it follows that

$$\xi_i(\zeta(t)) = \prod_{k=1}^{n_p} w_k^{\ell_{ik}}(\zeta(t)),$$

with $i \in \{1, \ldots, n_r\}$, $\ell_{ik} \in \{0,\ 1\}$, $k \in \{1, \ldots, n_p\}$. Consider that only s premise variables, with $s \leq n_p$, are available in real time for feedback. Define the subset $\mathbb{S} \subseteq \mathbb{K}_{n_p}$ as the set of indices of the premise variables available for feedback, where s_l denotes the elements of $\mathbb{S}$, $l = 1, 2, \ldots, n_s$, and n_s is the cardinality of $\mathbb{S}$. Consequently, only partial information from the membership functions $\xi_i(\zeta(t))$ is available for feedback. For each element $s_l \in \mathbb{S}$, define the following matrices:

$$\mathcal{K}_l = w_{s_l}^0 K_{s_l}^0 + w_{s_l}^1 K_{s_l}^1 \tag{16}$$

where $K_{s_l}^0$ and $K_{s_l}^1$ are matrices of appropriate dimensions. Since the matrices in (16) depend only on the functions $w_{s_l}^0$ and $w_{s_l}^1$, the next step is to relate $\mathcal{K}_l$ to the normalized membership functions $\xi_i(\zeta(t))$ of the system (4). Thus, from (6) it follows that

$$w_k^0(\zeta(t)) + w_k^1(\zeta(t)) = 1, \quad \forall k \in \mathbb{K}_{n_p}. \tag{17}$$

As a consequence, the matrices in (16) can be rewritten as

$$\begin{aligned}\mathcal{K}_l = &\prod_{k\in\mathbb{K}_{n_p}\setminus\mathbb{S}} \left(w_k^0(\zeta(t)) + w_k^1(\zeta(t))\right)\mathcal{K}_l \\ = &\prod_{k\in\mathbb{K}_{n_p}\setminus\mathbb{S}} \left(w_k^0(\zeta(t)) + w_k^1(\zeta(t))\right) w_{s_l}^0 K_{s_l}^0 \\ &+ \prod_{k\in\mathbb{K}_{n_p}\setminus\mathbb{S}} \left(w_k^0(\zeta(t)) + w_k^1(\zeta(t))\right) w_{s_l}^1 K_{s_l}^1.\end{aligned} \tag{18}$$

Now, define the index set $\mathbb{I}_{s_l}^0 = \{i \in \mathbb{K}_{n_r}$, such that $w_{s_l}^0$ composes the membership function $\xi_i(\zeta(t))\}$. Similarly, define $\mathbb{I}_{s_l}^1$. Considering (5) and (6), it follows from (18) that

$$\begin{aligned}\mathcal{K}_l = &\prod_{k\in\mathbb{K}_{n_p}\setminus\mathbb{S}} \left(w_k^0(\zeta(t)) + w_k^1(\zeta(t))\right) w_{s_l}^0 K_{s_l}^0 \\ &+ \prod_{k\in\mathbb{K}_{n_p}\setminus\mathbb{S}} \left(w_k^0(\zeta(t)) + w_k^1(\zeta(t))\right) w_{s_l}^1 K_{s_l}^1 \\ = &\left(\sum_{i\in\mathbb{I}_{s_l}^0} \xi_i(\zeta(t))\right) K_{s_l}^0 + \left(\sum_{i\in\mathbb{I}_{s_l}^1} \xi_i(\zeta(t))\right) K_{s_l}^1.\end{aligned} \tag{19}$$

By rearranging the terms, (19) can be rewritten as follows:

$$\mathcal{K}(\xi) = \sum_{i=1}^{n_r} \xi_i(\zeta(t)) K_i, \tag{20}$$

with

$$K_i = \sum_{(s_l, \ell_{is_l})\in\mathbb{J}_i} K_{s_l}^{\ell_{is_l}}, \tag{21}$$

where $\mathbb{J}_i = \{(s_l, \ell_{is_l})\}$, denote the set of pairs such that the element $w_{s_l}^{\ell_{is_l}}$ is part of the product in (5) that composes the membership function $\xi_i(\zeta(t))$, with $i \in \mathbb{K}_{n_r}$, $l \in \mathbb{K}_{n_s}$, and $s_l \in \mathbb{S}$. Therefore, the control input (7) can be designed considering only the available premise variables, with the matrix $K(\xi)$ defined according to (16), (20), and (21).

Example 1: Consider a system (4) such that $n_p = 3$, $n_r = 8$ and the available premise variables are $\mathbb{S} = \{1, 3\}$. As a consequence, $n_s = 2$, $s_1 = 1$, $s_2 = 3$,

$$\mathcal{K}_1 = w_1^0 K_1^0 + w_1^1 K_1^1, \quad \mathcal{K}_2 = w_3^0 K_3^0 + w_3^1 K_3^1. \tag{22}$$

By (5), (6) and the binary representation [2]:

$$i = 1 + \ell_{i1} \times 2^0 + \ell_{i2} \times 2^1 + \ell_{i3} \times 2^2$$

with $i \in \mathbb{K}_{n_r}$ and $\ell_{ik} \in \{0,\ 1\}$, it follows that

$$\begin{aligned}\xi_1 &= w_1^0 w_2^0 w_3^0, & \xi_5 &= w_1^0 w_2^0 w_3^1, \\ \xi_2 &= w_1^1 w_2^0 w_3^0, & \xi_6 &= w_1^1 w_2^0 w_3^1, \\ \xi_3 &= w_1^0 w_2^1 w_3^0, & \xi_7 &= w_1^0 w_2^1 w_3^1, \\ \xi_4 &= w_1^1 w_2^1 w_3^0, & \xi_8 &= w_1^1 w_2^1 w_3^1.\end{aligned} \tag{23}$$

Considering the steps (18) and (19), matrices (22) are rewritten as

$$\begin{aligned}\mathcal{K}_1 = &(\xi_1 + \xi_3 + \xi_5 + \xi_7) K_1^0 \\ &+ (\xi_2 + \xi_4 + \xi_6 + \xi_8) K_1^1,\end{aligned} \tag{24}$$

$$\begin{aligned}\mathcal{K}_2 = &(\xi_1 + \xi_2 + \xi_3 + \xi_4) K_3^0 \\ &+ (\xi_5 + \xi_6 + \xi_7 + \xi_8) K_3^1.\end{aligned} \tag{25}$$

Finally, by (20) and (21) it is obtained

$$\mathcal{K}(\xi) = \sum_{i=1}^{8} \xi_i(\zeta(t)) K_i \tag{26}$$

where

$$\begin{aligned}&K_1 = K_1^0 + K_3^0,\ K_2 = K_1^1 + K_3^0,\ K_3 = K_1,\ K_4 = K_2, \\ &K_5 = K_1^0 + K_3^1,\ K_6 = K_1^1 + K_3^1,\ K_7 = K_5,\ K_8 = K_6.\end{aligned}$$

Inspired by [4, Theorem 10] and taking into account the available premise variables and matrices (16), (20), and (21), next, a sufficient LMI-based condition for $\mathfrak{D}$-region constraint of local models from the system (8) into the region of [4, Fig. 1], is proposed.

Theorem 1: Given the subset $\mathbb{S} \subset \mathbb{K}_p$, scalars β, ϕ, r, q, γ, $\varepsilon > 0$, matrices $\mathfrak{N}$, $\mathfrak{M}$ defined in (10) and (11), respectively, and matrices $L \in \mathbb{R}^{n_x - n_y \times n_y}$, $Q \in \mathbb{R}^{n_x \times n_x - n_y}$, such that $C_y Q = 0$. If there exist matrices $F_{11} \in \mathbb{R}^{n_x - n_y \times n_x - n_y}$, $F_{12} \in \mathbb{R}^{n_x - n_y \times n_y}$, $F_{22} \in \mathbb{R}^{n_y \times n_y}$, $Y_{Rs_l}^0 \in \mathbb{R}^{n_u \times n_y}$ and $Y_{Rs_l}^1 \in \mathbb{R}^{n_u \times n_y}$, such that the following LMI conditions hold

$$\Psi_{ii} \prec 0, \tag{27}$$

$$\frac{2}{n_r - 1}\Psi_{ii} + \Psi_{ij} + \Psi_{ji} \prec 0, \tag{28}$$

with

$$\Psi_{ij} = \begin{bmatrix} \Psi_{ij}^{(1,1)} & * \\ \Psi_{ij}^{(2,1)} & -\varepsilon I \otimes \mathcal{H}e(F) \end{bmatrix},$$

$$\Psi_{ij}^{(1,1)} = \mathfrak{N} \otimes P + \mathcal{H}e\big(\mathfrak{M} \otimes (A_i F + B_i Y_j)\big),$$
$$\Psi_{ij}^{(2,1)} = \mathfrak{M} \otimes (P - F) + \varepsilon I \otimes \big(F^T A_i^T + K_j^T Y_i^T\big),$$
$$F = \begin{bmatrix} Q & R \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} \\ 0 & F_{22} \end{bmatrix} \begin{bmatrix} Q^T \\ R^T \end{bmatrix},$$
$$Y_j = Y_{Rj} R^T, \quad R = {C_y}^{\dagger} + QL, \quad Y_{Rj} = \sum_{(s_l, \ell_{js_l}) \in \mathbb{J}_j} Y_{Rs_l}^{\ell_{js_l}},$$

for all $i, j \in \mathbb{K}_{n_r}$, $i \neq j$, $l \in \mathbb{K}_{n_s}$. Then, the T-S model (4) is $\mathfrak{D}$-region constraint by the PDC control law (7) with $K_j = Y_{Rj} F_{22}^{-1}$.

Proof: If LMIs (27) and (28) hold, then

$$\Psi(\xi) = \begin{bmatrix} \Psi(\xi)^{(1,1)} & * \\ \Psi(\xi)^{(2,1)} & -\varepsilon I \otimes \mathcal{H}e\big(F\big) \end{bmatrix} \prec 0 \tag{29}$$

where

$$\Psi(\xi)^{(1,1)} = \mathfrak{N} \otimes P + \mathcal{H}e\big(\mathfrak{M} \otimes (A(\xi)F + B(\xi)Y(\xi))\big),$$
$$\Psi(\xi)^{(2,1)} = \mathfrak{M} \otimes \big(P - F\big) + \varepsilon I \otimes \big(F^T A(\xi)^T + Y(\xi)^T B(\xi)^T\big)$$

$[A(\xi) \quad B(\xi) \quad Y(\xi)] = \sum_{i=1}^{r} \xi_i(\zeta(t))[A_i \quad B_i \quad Y_i]$. By (29), $F + F^T \succ 0$, ensuring that F is nonsingular. Taking into account (14), it follows that

$$Y_j = K_j C_y F, \tag{30}$$

for all $j \in \mathbb{K}_{n_r}$. Consider two arbitrary matrices U and V with appropriate dimensions and $A_\xi(\xi) = \big(A(\xi) + B(\xi)Y(\xi)F^{-1}\big)$. One may introduce the null terms:

$$\mathcal{H}e\big(\mathfrak{M} \otimes A_\xi(\xi))U\big) - \mathcal{H}e\big(\mathfrak{M} \otimes A_\xi(\xi)U\big) = 0, \tag{31}$$
$$\mathcal{H}e\big(I \otimes A_\xi(\xi)VA_\xi(\xi)^T\big) - \mathcal{H}e\big(I \otimes A_\xi(\xi)VA_\xi(\xi)^T\big) = 0,$$

By $\mathfrak{N} \otimes P + \mathcal{H}e\big(\mathfrak{M} \otimes A_\xi(\xi)P\big) \prec 0$ summing with (31), the inequality is equivalent to:

$$\begin{aligned} &\mathfrak{N} \otimes P + \mathcal{H}e(\mathfrak{M} \otimes A_\xi(\xi)U) + \\ &\mathcal{H}e\big(\mathfrak{M} \otimes A_\xi(\xi)P - \mathfrak{M} \otimes A_\xi(\xi)U + I \otimes A_\xi(\xi)V^T A_\xi(\xi)^T\big) \\ &-I \otimes \mathcal{H}e\big(A_\xi(\xi)VA_\xi(\xi)^T\big) \prec 0, \end{aligned} \tag{32}$$

that is to say:

$$\begin{aligned} &\mathfrak{N} \otimes P + \mathcal{H}e(\mathfrak{M} \otimes A_\xi(\xi)U) \\ &+\mathcal{H}e\Big((I \otimes A_\xi(\xi))\big(\mathfrak{M} \otimes (P - U) + I \otimes V^T A_\xi(\xi)^T\big) \\ &+(I \otimes A_\xi(\xi))\big(-I \otimes \mathcal{H}e(V)\big)(I \otimes A_\xi(\xi)^T)\Big) \prec 0, \end{aligned} \tag{33}$$

or, equivalently,

$$\begin{bmatrix} I \\ I \otimes A_\xi(\xi) \end{bmatrix}^T \begin{bmatrix} \Psi(\xi)^{(1,1)} & * \\ \Psi(\xi)^{(2,1)} & -I \otimes \mathcal{H}e\big(V\big) \end{bmatrix} \begin{bmatrix} I \\ I \otimes A_\xi(\xi) \end{bmatrix} \prec 0 \tag{34}$$

where

$$\Psi(\xi)^{(1,1)} = \mathfrak{N} \otimes P + \mathcal{H}e(\mathfrak{M} \otimes A_\xi(\xi)U),$$
$$\Psi(\xi)^{(2,1)} = \big(\mathfrak{M} \otimes (P - U) + I \otimes V^T A_\xi(\xi)^T\big).$$

Inequality (34) holds if:

$$\begin{bmatrix} \Psi(\xi)^{(1,1)} & * \\ \Psi(\xi)^{(2,1)} & -I \otimes \mathcal{H}e\big(V\big) \end{bmatrix} \prec 0, \tag{35}$$

or, equivalently (29) with $U = F$, $V = \varepsilon F$ and [17, Theorem 2.2].

Remark 1: For practical implementation, PDC control input (7) needs to be rewritten as

$$u(t) = \sum_{l=1}^{n_s} \left[w_{s_l}^0 Y_{s_l}^0 + w_{s_l}^1 Y_{s_l}^1\right] F_{22}^{-1} y(t). \tag{36}$$

V. Control design and simulation

Two illustrative examples are presented in this section. The controllers were developed in the MATLAB/Simulink® environment, employing the SDPT3 solver [18] in conjunction with the YALMIP toolbox [19].

Example 2: Let us consider the T–S model with $n_p = 2$, $n_r = 4$, and $\mathbb{S} = 1$. Consequently, the available premise function is defined as $w_1^0(\boldsymbol{\zeta}(t)) = \frac{1-\sin(x1)}{2}$ and $w_1^1(\boldsymbol{\zeta}(t)) = 1 - w_1^0(\boldsymbol{\zeta}(t))$, and vertices given by

$$\begin{aligned} A_1 &= \begin{bmatrix} 1+a & 0 & 1 & -0.1 \\ 1 & -2 & 0 & 0 \\ 1 & a^2 & -0.3 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}, \; B_1 = \begin{bmatrix} 0.5 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 1-a & 0 & b^2 & -0.1 \\ 1 & -2 & 0 & 0 \\ 1 & a^2 & -0.3 & 0 \\ 0 & 0 & b & -1 \end{bmatrix}, \; B_2 = \begin{bmatrix} 0.5 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} 1+a & 0 & 1 & -0.1 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & -0.3 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}, \; B_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \\ A_4 &= \begin{bmatrix} 1-a & 1 & b^2 & -0.1 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & -0.3 & 0 \\ 0 & 0 & b & -1 \end{bmatrix}, \; B_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \end{aligned} \tag{37}$$

Considering the system vertices defined in (37), over the parameter range (a, b) with $a \in [0, 2]$ and $b \in [0, 10]$, the LMIs (27)–(28) from Theorem 1 were solved by setting $\beta = 5$, $\phi = \pi/17$, $r = 6$, $q = -3$, $\gamma = 6$, and $\varepsilon = 0.01$. Additionally, the matrices $C_y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, $Q = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T$, and $L = \begin{bmatrix} -0.04 & -0.3 \\ 0.04 & 0.6 \end{bmatrix}$ were considered. The results can be observed in Fig. 1.

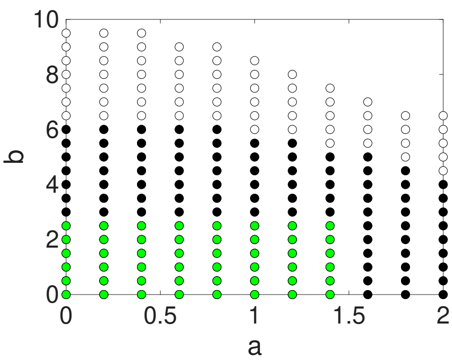

Fig. 1. Feasibility region for a and b considering Theorem 1: using the decomposition proposed in [12] (●); the decomposition in [11, Theorem 1] (●, ●); and the decomposition based on Lemma 1 (●, ●, ○).

Fig. 1 shows the simulation results of Theorem 1 using the matrix decompositions in [11] and [12] for the control input (7) with a fixed gain ($u(t) = Ky(t)$), and the proposed method with gain (21). The decomposition based on Lemma 1 yields a larger feasibility region than existing approaches.

Example 3: Consider the quarter-car active suspension system on the Quanser® platform [5, Fig. 3]. The nonlinear dynamics of the active suspension system [20], [21]:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\dfrac{k_s}{M_s} & -\dfrac{b_s}{M_s} & 0 & \dfrac{b_s}{M_s} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_s}{M_{us}} & \dfrac{b_s}{M_{us}} & f_{43}(\zeta(t)) & -\dfrac{(b_s+b_{us})}{M_{us}} \end{bmatrix} x(t)$$

$$+ \begin{bmatrix} 0 \\ \dfrac{k_{\text{fault}}}{M_s} \\ 0 \\ -\dfrac{k_{\text{fault}}}{M_{us}} \end{bmatrix} u(t),\ f_{43}(\zeta(t)) = -\frac{k_{us_0}(1+\Delta k_{us}|z_{us}-z_r|)}{M_{us}},$$

$$x(t) = \begin{bmatrix} z_s(t) - z_{us}(t) \\ \dot{z}_s(t) \\ z_{us}(t) - z_r(t) \\ \dot{z}_{us}(t) \end{bmatrix},\ y(t) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}^T x(t). \tag{38}$$

The system parameters are given in [21, Table 1]. Accordingly, the domain $\mathscr{D}$ for $f_{43}(\zeta(t))$, $k_{\text{fault}}(t)$, and M_s is defined as: $\mathscr{D} = \Big\{\zeta(t) = \begin{bmatrix} x(t)^T & \Delta k_{us} & k_{\text{fault}} & M_s \end{bmatrix}^T \in \mathbb{R}^7 : -0.02 \leq z_{us} - z_r \leq 0.02,\ 0 \leq \Delta k_{us} \leq 60, 0.8 \leq k_{\text{fault}}(t) \leq 1,\ 1.455 \leq M_s \leq 2.45\Big\}$. Then, from (38), domain $\mathscr{D}$ and parameters [21, Table 1], the following local models are

$$A_1 = A_3 = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -367.35 & -3.0612 & 0 & 3.0612 \\ 0 & 0 & 0 & 1 \\ 900 & 7.5 & -3000 & -12.5 \end{bmatrix},$$

$$A_2 = A_4 = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -367.35 & -3.0612 & 0 & 3.0612 \\ 0 & 0 & 0 & 1 \\ 900 & 7.5 & -2500 & -12.5 \end{bmatrix},$$

$$A_5 = A_7 = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -618.56 & -5.1546 & 0 & 5.1546 \\ 0 & 0 & 0 & 1 \\ 900 & 7.5 & -3000 & -12.5 \end{bmatrix},$$

$$A_6 = A_8 = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -618.56 & -5.1546 & 0 & 5.1546 \\ 0 & 0 & 0 & 1 \\ 900 & 7.5 & -2500 & -12.5 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} 0 \\ 0.32653 \\ 0 \\ -0.8 \end{bmatrix},\ B_3 = B_4 = \begin{bmatrix} 0 \\ 0.40816 \\ 0 \\ -1 \end{bmatrix},$$

$$B_5 = B_6 = \begin{bmatrix} 0 \\ 0.54983 \\ 0 \\ -0.8 \end{bmatrix},\ B_7 = B_8 = \begin{bmatrix} 0 \\ 0.68729 \\ 0 \\ -1 \end{bmatrix},$$

$$C_y = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{39}$$

It is worth emphasizing that the matrix L plays a crucial role in the numerical feasibility of the proposed LMIs. To address this issue, a two-stage design strategy, as proposed in [22], was adopted. In the first stage, the LMIs (27)–(28) of Theorem 1 were solved by setting $\beta = 5$, $\phi = \pi/17$, $r = 80$, $q = -3$, $\gamma = 6$, and $\varepsilon = 0.01$, while considering $C_y = I$ and $L = 0$, which corresponds to a state feedback configuration. Under these conditions, the following matrix was obtained:

$$P_x = \begin{bmatrix} 0.2236 & -0.3562 & 0.0012 & -0.0032 \\ -0.3562 & 6.1115 & 0.0383 & 0.8800 \\ 0.0012 & 0.0383 & 0.0036 & -0.0387 \\ -0.0032 & 0.8800 & -0.0387 & 5.4283 \end{bmatrix} \times 10^3.$$

In the second stage, the output feedback gain was designed. For this purpose, the following matrix was selected:

$$Q = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}^T, \tag{40}$$

and the matrix L was computed according to the procedure described in [22], yielding

$$\begin{aligned} L &= Q^\dagger P_x^{-1} C_y^T \left(C_y P_x^{-1} C_y^T\right)^{-1} \\ &= \begin{bmatrix} -0.0065 & 0.1012 \\ 0.0226 & -1.3151 \end{bmatrix}. \end{aligned} \tag{41}$$

Finally, for the output feedback controller design, we consider the system vertices in (39), with $C_y = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, the matrices Q and L given in (40) and (41), respectively, the LMIs (27)–(28) from Theorem 1, and the same design parameters declared previously. Under these conditions, the following matrices were obtained:

$$Y_{R1} = \begin{bmatrix} -6.4944 \\ -0.0743 \end{bmatrix}^T \times 10^7,\ Y_{R2} = \begin{bmatrix} -6.3716 \\ -0.0499 \end{bmatrix}^T \times 10^7,$$

$$F_{22} = \begin{bmatrix} 1.0481 & 0.0069 \\ 0.0079 & 0.0007 \end{bmatrix} \times 10^6. \tag{42}$$

Fig. 2 illustrates the migration of poles of linearized models of the closed-loop models along the output state trajectories, for the initial condition $x(0) = \begin{bmatrix} 0.02 & 0 & 0 & 0 \end{bmatrix}$.

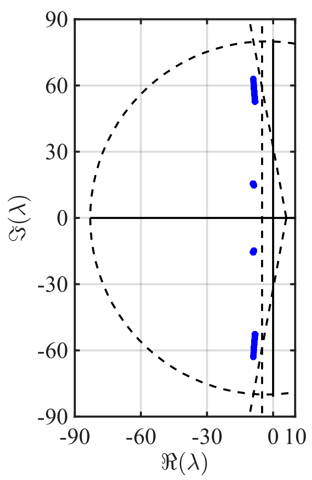

Fig. 2. Pole placement for the local models of the system (39).

The simulation results are shown in Figs. 3 and 4, in which a square-wave input with an amplitude of 0.02 m is applied to the road profile. The analysis is carried out using the nonlinear model of the active suspension system given in (38). An output feedback control law, defined in (36), is

$$u(t) = \sum_{l=1}^{n_s} \left[w_{s_l}^0 Y_{Rs_l}^0 + w_{s_l}^1 Y_{Rs_l}^1 \right] F_{22}^{-1} y(t).$$

From 0 to 9 s, the system operates in open loop, without any control action, as indicated by the vertical green line. At $t = 9$ s, the controller is activated, and the system transitions to closed-loop operation. Finally, from 18 to 30 s, the system operates under energy losses of up to 20%. Fig. 3 shows a significant attenuation of transient oscillations, with reduced overshoot and settling time, indicating improved damping and faster convergence.

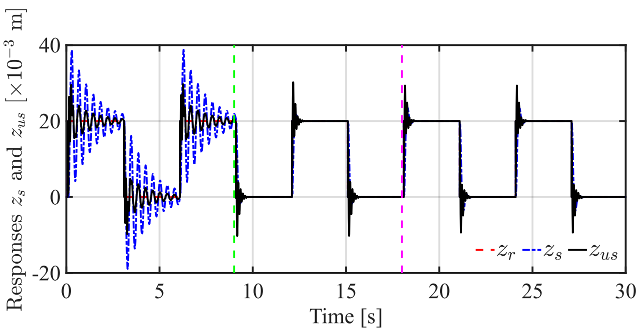

Fig. 3. System responses and track profile on active suspension simulation.

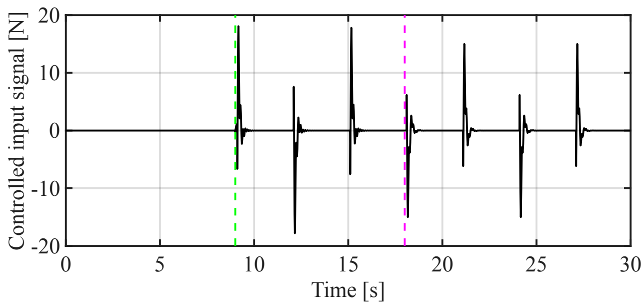

Fig. 4. Controlled input signal on active suspension simulation.

VI. CONCLUSION

This paper proposes a PDC output feedback control strategy under $\mathfrak{D}$-region constraints for uncertain nonlinear systems represented by Takagi–Sugeno fuzzy models. The $\mathfrak{D}$-region is enforced by prescribing a stability region that guarantees a desired decay rate for the closed-loop system. The LMIs are derived by extending a state feedback result to the output feedback case via a matrix decomposition that exploits the structural properties of the system output matrix, along with a premise-selection strategy that enables the design and implementation of a PDC controller. The obtained results demonstrate the effectiveness of the proposed strategy, successfully achieving pole placement within a prescribed region. Future work includes investigating alternative Lyapunov functions and implementing the proposed technique in the actual system module.

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