ISurvey: An R Toolkit for Interval-Valued Survey Data Analysis

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***Abstract*—Recent years have seen growing interest in interval-valued (IV) survey response formats as an alternative to conventional single-valued (SV) scales, as they allow respondents to express uncertainty and imprecision as ranges. While IV data analysis methods have advanced, software support for applied IV survey analysis remains limited. To address this gap, this paper presents `ISurvey`, an R toolkit for IV survey data analysis, designed for researchers and practitioners with diverse backgrounds. `ISurvey` supports datasets containing IV, SV, and single-choice items. It provides descriptive statistics under the Fréchet framework, visual descriptives, and survey psychometric assessment methods such as IV correlation for convergent validity and IV Cronbach's α for reliability. The toolkit also implements IV extensions of diagnostic methods used for actionable prioritization and benchmarking in survey research, including gap analysis, importance–performance analysis, and the customer satisfaction index. For illustration, we present a complete data analysis workflow using a UK rail customer survey, in which IV and SV responses were collected via a between-subject design. This provides a step-by-step tutorial that guides users from raw exports (from DECSYS, an IV survey platform) to analysis outputs ready for reporting and decision-making.**

***Index Terms*—Intervals, Surveys, R Toolkit, Fuzzy sets, Uncertainty**

I. Introduction

Surveys are widely used to capture human perceptions, judgments, and evaluations across domains such as marketing, customer experience, the public sector, and healthcare [1]–[3]. Traditional response formats, such as Likert and visual analogue scales, are typically single-valued (SV), requiring respondents to select a single point on a scale (Fig. 1a). However, this can restrict respondents' authentic expression, particularly when facing question complexity (e.g., survey items like overall satisfaction that involve multiple touch-points) or inherent vagueness in their own evaluations, where ratings may lie imprecisely between linguistic labels or be more naturally expressed as a range [3]–[5].

The IV response format was proposed and validated as an alternative, allowing a respondent to express a range (i.e., provide an interval) rather than a single point, as illustrated in Fig. 1b [3]. IV response formats have been deployed across diverse fields, including cybersecurity, consumer research, healthcare, and environmental management [6]–[10].

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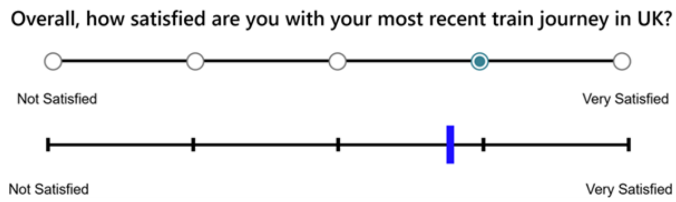
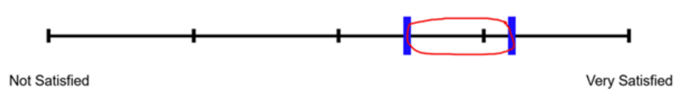


(a) Likert and visual analogue scale



(b) IV response formats

Fig. 1: Illustrative example of how imprecision and uncertainty are (or are not) expressed in different response formats.

To facilitate its broader adoption in practice, software support for IV survey elicitation has developed in recent years. Platforms such as Qualtrics and SurveyJS can be customized to elicit IV responses; more recently, DECSYS provides an alternative that natively supports IV surveys without programming, alongside SV, single-choice, and free-text items [11]. DECSYS captures IV responses e.g., via a drawing interaction (Fig. 1b). It also exports survey data in structured formats such as `.csv` and `.json` for downstream analysis (Table I).

TABLE I: Examples of selected response modes in DECSYS.

Partic-ipant	Page_ Name	Response Type	Ellipse Scale_min	Ellipse Scale_max	Discrete Scale	Choose One
1	Q1	Ellipse	3.2	4.5	–	–
1	Q2	Discrete	–	–	4	–
1	Gender	Choose One	–	–	–	Male

Beyond data elicitation, IV survey responses require analysis methods tailored to interval data, and a broad literature has developed over the last decades. This includes descriptive statistics for central tendency and dispersion (e.g., mean and variance) [12], agreement-based fuzzy modeling for visual summarization (e.g., the Interval Agreement Approach (IAA) [13] and its IV summaries via interval-valued defuzzification (IVD) [14]–[18]), and inferential methods such as regression, classification, and hypothesis testing [19]–[22].

Despite these advances, IV data analysis presents structural and computational challenges in survey contexts. In particular, when intervals are interpreted as unknown-but-bounded values, variance is expressed in terms of tight bounds, and computing these bounds is NP-hard in general [12]. This complicates direct extensions of variance-based measures [19], including correlation and Cronbach's α – methods essential to psychometric assessment in conventional survey workflows.

To address this, a practical approach is to adopt the Fréchet framework, which treats IV responses as elements of a metric space and defines mean and variance once a distance (d) is specified [19]. This supports IV extensions of variance-based measures used in psychometric assessment, including Cronbach's α based on θ distance (d_θ) [19] and correlation based on Hausdorff (H) distance (d_H) [23].

While these methodological advances address key analytical challenges, their adoption in applied workflows depends on accessible software support. Several symbolic data analysis (SDA) packages [24], [25] support general IV analysis such as clustering and regression, but do not support survey-oriented methods such as psychometric assessment. In contrast, `IntervalQuestionStat` (IQS) is a pioneering publicly available toolkit specifically designed for IV survey analysis [26]; the methods it implements are summarized in Table II. However, it does not yet incorporate recent H-based developments for descriptive statistics and correlation, nor visual descriptives such as IAA and IVD.

We therefore develop `ISurvey`, an R toolkit designed to enable researchers from diverse backgrounds to conduct IV survey analysis within conventional workflows. `ISurvey` complements IQS by incorporating recent H-based methods and visual descriptives. In addition, `ISurvey` incorporates IV extensions of diagnostic methods that are essential to conventional survey practice, supporting actionable prioritization and benchmarking that underpin real-world decision-making. These include the three most widely used methods: gap analysis, importance–performance analysis (IPA), and the customer satisfaction index (CSI) [27]–[29]. The methods implemented in `ISurvey` are summarized in Table II, and the toolkit is source-available at https://github.com/Carina-YuZhao to encourage wider adoption and collaboration.

TABLE II: Methods Comparison of IQS and ISurvey

Component	Methods	IQS	ISurvey
Descriptives	Fréchet statistics (θ / H)	✓ / –	✓ / ✓
	RFH, IAA, IVD	–	✓
Psychometric	Cronbach's α (θ / H)	✓ / –	✓ / ✓
	Correlation (midpoint, arithmetic, θ, H)	–	✓
Diagnostics	Gap analysis, IPA, CSI	–	✓

This paper presents a complete IV survey analysis workflow as a step-by-step tutorial, from DECSYS exports to `ISurvey` outputs. In practice, the DECSYS exports can be replaced with IV data from any other source. The workflow is demonstrated using a real-world UK rail passenger survey case study, with IV and SV responses collected via DECSYS for comparison.

The remainder of this paper is organized as follows. Section II introduces the interval representations and methods implemented in the toolkit. Section III presents a workflow demonstration with detailed code and visual results. Section IV concludes and outlines future work.

II. Background

This section introduces IV data representations and reviews the methods supported by `ISurvey` thus far, including descriptive statistics, visual descriptives, psychometric assessment methods, and diagnostic methods for applied survey analysis.

A. Interval-Valued Response

An IV response is represented as a continuous interval

$$\overline{X} = [X^-, X^+], \qquad X^- \leq X^+, \tag{1}$$

where X^- and X^+ denote the lower and upper bounds, respectively. Equivalently, $\overline{X}$ can be characterized by its midpoint and spread:

$$\mathrm{m}(\overline{X}) = \frac{X^- + X^+}{2}, \qquad \mathrm{s}(\overline{X}) = \frac{X^+ - X^-}{2}. \tag{2}$$

A discontinuous (multi-segment) interval is defined as the union of n pairwise disjoint continuous sub-intervals, $\overline{\overline{X}} = \bigcup_{i=1}^{n} \overline{X_i}$. When $n = 1$, $\overline{\overline{X}} = \overline{X}$.

B. Interval Distances and the Fréchet Framework

Unlike SV data, IV data lack a natural arithmetic structure, necessitating the definition of distance metrics to quantify discrepancies between them. `ISurvey` implements two widely used interval distances: d_θ, which offers computational convenience through its midpoint-spread decomposition [19], and d_H, which provides a geometric interpretation and accommodates discontinuous intervals [23].

For intervals $\overline{X}$ and $\overline{Y}$, d_θ is defined as

$$d_\theta(\overline{X}, \overline{Y}) = \sqrt{\left(\mathrm{m}(\overline{X}) - \mathrm{m}(\overline{Y})\right)^2 + \theta \left(\mathrm{s}(\overline{X}) - \mathrm{s}(\overline{Y})\right)^2}, \tag{3}$$

where $\theta \geq 0$ controls the relative contribution of location (midpoint) and imprecision (spread).

For intervals $\overline{\overline{X}}$ and $\overline{\overline{Y}}$, d_H is defined as

$$d_H(\overline{\overline{X}}, \overline{\overline{Y}}) = \max \left\{ \sup_{x \in \overline{\overline{X}}} \inf_{y \in \overline{\overline{Y}}} |x - y|, \ \sup_{y \in \overline{\overline{Y}}} \inf_{x \in \overline{\overline{X}}} |x - y| \right\}. \tag{4}$$

When $\overline{\overline{X}} = \overline{X}$ and $\overline{\overline{Y}} = \overline{Y}$, this reduces to

$$d_H(\overline{X}, \overline{Y}) = \max\left\{ |X^- - Y^-|, |X^+ - Y^+| \right\}. \tag{5}$$

Given a distance metric d, the Fréchet framework generalizes the classical mean to IV data: the sample Fréchet mean of n IV responses $\overline{\boldsymbol{X}} = \{\overline{X}_i\}_{i=1}^n$ is defined as the interval that minimizes the average squared distance to all responses:

$$\overline{X}^* = \arg\min_{\overline{X}} \frac{1}{n} \sum_{i=1}^{n} d^2(\overline{X}, \overline{X}_i), \tag{6}$$

The corresponding sample Fréchet variance is

$$\mathrm{Var}^*(\overline{\boldsymbol{X}}) = \frac{1}{n} \sum_{i=1}^{n} d^2(\overline{X}_i, \overline{X}^*). \tag{7}$$

Under d_θ, the Fréchet mean coincides with the arithmetic (Aumann) mean of the interval endpoints [19]. Under d_H, the Fréchet mean is not necessarily unique; recent work addresses this by restricting to continuous intervals and fixing the interval width while optimizing the midpoint [23].

C. Interval Agreement Approach

The Interval Agreement Approach (IAA) models IV data as a fuzzy set (FS) and provides an overview of response distributions, analogous to Relative Frequency Histograms (RFH) for SV data [13]. Given an IV dataset $\overline{\boldsymbol{X}} = \{\overline{X}_i\}_{i=1}^n$ with $\overline{X}_i = [X_i^-, X_i^+]$, the IAA method constructs a FS A with membership function $\mu_A(x)$:

$$\mu_A(x) = \frac{1}{n}\sum_{i=1}^{n} \mu_{\overline{X}_i}(x), \tag{8}$$

where

$$\mu_{\overline{X}_i}(x) = \begin{cases} 1, & \text{if } X_i^- \le x \le X_i^+, \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

D. Interval-Valued Defuzzification

While IAA FSs provide visual summaries, comparing many FSs can be difficult in practice, particularly in surveys with many aspects/questions. Defuzzification methods such as the centroid reduce a FS to a single point to support comparison, but this discards both uncertainty and distributional information. Interval-Valued Defuzzification (IVD) instead summarizes a FS as an interval by aggregating its α-cuts, retaining uncertainty via interval width [14]–[16]. An α-cut of an FS A is defined as the set $\{x \mid \mu_A(x) \ge \alpha\}$. The output can be a continuous interval $\overline{X}$ or a discontinuous interval $\overline{\overline{X}}$.

IVD methods include IV Averaging Level Cuts (IALC) [16], Nested Union-based (NU), Disjoint Union-based (DU) [17], [30], and their weighted variants (NUW, DUW) that weight α-cuts by α. IALC always returns a continuous interval, whereas NU- and DU-based methods may produce discontinuous intervals. This preserves multi-modal structure and makes the presence or absence of group consensus explicit.

E. Psychometric Assessment Methods

Psychometric assessment is commonly a necessary component of survey analysis, typically covering both validity and reliability. Validity includes several forms, such as content validity and construct validity, which examine whether a survey instrument appropriately represents and measures the intended construct. In this work, we focus on convergent validity, which is commonly assessed by examining whether two measures intended to capture the same construct are positively associated. Accordingly, when a new measurement method is introduced, convergent validity is evaluated by examining its correlation with an established measure.

The correlation coefficient is defined as

$$\rho(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\,\mathrm{Var}(Y)}}. \tag{10}$$

For IV data, methods to establish correlation include: (i) midpoint-based Pearson correlation; (ii) arithmetic correlation combining midpoint and width correlations [31]; and (iii) Fréchet-based correlation, computed under d_H [23] or d_θ using the variance and covariance provided in [19].

Reliability refers to the extent to which a survey instrument produces consistent measurements and is most commonly assessed using Cronbach's α. For a survey with k items, Cronbach's α evaluates internal consistency by comparing the sum of item variances to the variance of the total score:

$$\alpha = \frac{k}{k-1}\left(1 - \frac{\sum_{j=1}^{k}\mathrm{Var}(X_j)}{\mathrm{Var}\left(\sum_{j=1}^{k} X_j\right)}\right). \tag{11}$$

For IV data, Cronbach's α has been extended by substituting the θ-Fréchet variances into (11), while preserving key mathematical properties of the classical Cronbach's α [19]. Similarly, with the H-Fréchet variance provided in [23], an H Cronbach's α can be obtained by substituting it into (11); however, its theoretical properties remain to be studied.

F. Diagnostic Methods

Customer research uses diagnostic methods to prioritize service attributes, among which gap analysis, IPA, and CSI are widely used in practice.

Gap analysis assesses service shortfalls by computing $\mathrm{Gap} = P - E$, where E and P denote mean expectation and perception; negative gaps indicate service shortfalls [27].

IPA maps attributes on a grid of mean importance (y-axis) versus mean performance (x-axis), with the overall means defining four quadrants for prioritization [28].

CSI provides an importance-weighted satisfaction index, commonly used for benchmarking over time or across providers [29]:

$$\mathrm{CSI} = \frac{\sum_{j=1}^{k} I_j \cdot S_j}{\sum_{j=1}^{k} I_j}, \tag{12}$$

where I_j and S_j are the mean importance and satisfaction for attribute j.[1]

Through interval arithmetic (such as subtraction and weighted aggregation), `ISurvey` extends all three methods to IV data and provides comparisons with their conventional SV counterparts, as detailed in Section III.

III. Workflow Demonstration

Survey analysis typically follows a structured workflow–descriptive summarization and visualization, psychometric assessment, and diagnostic analysis–as reviewed in Section II. `ISurvey` extends this workflow for IV data. This section briefly summarizes the toolkit design and demonstrates an integrated workflow for analyzing a between-subject survey containing both SV and IV responses, from data loading through analysis to export. While multiple method options are supported at each stage, the choices used here are primarily intended to demonstrate an IV workflow alongside a traditional SV workflow, and to illustrate a range of toolkit features. The

[1] In practice, importance and satisfaction are often reused as proxies for expectation and perception/performance, as in Section III.

source code and data for this demonstration are available on GitHub at https://github.com/Carina-YuZhao.

A. Toolkit Design

ISurvey is designed to handle both IV and SV responses across all stages of the survey analysis workflow, supporting SV-only, IV-only, IV–SV, and IV–IV analyses, as well as comparison and visualization. When multiple data sources are provided, items are matched by Page Name as identifiers, and response types (IV or SV) are detected automatically, allowing IV and SV responses sharing the same Page Name to be processed together.

B. Study Design

The data used for demonstration is based on a rail customer study conducted in the United Kingdom. The study was designed as a between-subject experiment, where participants completed either an SV survey ($n = 85$ participants) or an IV survey ($n = 81$ participants). The survey consisted of 14 service aspects, with participants rating both importance and satisfaction for each; in this case study, the corresponding items were labeled I1–I14 and S1–S14 as Page Names. Chi-square tests confirmed that the two groups were comparable across key demographic variables. Ethical approval was granted by the University of Nottingham.

C. Data Load

load_data() imports IV and SV responses from CSV files, such as those exported from DECSYS, and standardizes the raw imports into a consistent format for downstream analysis. By default, a built-in schema matches the standard DECSYS export format. For non-DECSYS datasets, a custom schema can map dataset-specific column names.

```
iv_data <- load_data("Rail_user_survey_interval.csv")
sv_data <- load_data("Rail_user_survey_5-point.csv")
Sat <- paste0("S", 1:14)  # satisfaction items
Imp <- paste0("I", 1:14)  # importance items
LABELS <- c("1 Journey Info", "2 Wait Area", ...)
```

The same Page Names are used across both IV and SV exports, enabling automatic item matching.

list_choices() inspects demographic labels stored as choose_one responses.

```
list_choices(iv_data)
#$Device: "Desktop"  "Mobile phone"  "Tablet"
#$Gender: "Female"  "Male"  "Other"
```

filter_by_choice() supports subgroup analysis by filtering respondents based on selected choice values and returning the corresponding sub-dataset.

```
iv_fd <- filter_by_choice(iv_data, c("Female", "Desktop"))
```

D. Descriptive Statistics

calc_descriptives() computes item-wise descriptive summaries for IV and SV data.

```
# method: "theta" (default), "hausdorff", "both"
df_S <- calc_descriptives(iv_data, sv_data, items=Sat,
     method="both")
df_I <- calc_descriptives(iv_data, sv_data, items=Imp,
     method="both")
```

```
> df_I$theta["I1",]  # IV theta: [3.76, 4.24]
> df_I$H["I1",]      # IV H:     [3.79, 4.28]
> df_I$sv["I1"]      # SV:       4.18
```

plot_means() visualizes item-wise mean summaries. By default, all available means (θ, H, SV) are displayed; specific subsets can be selected via show. Here, Fig. 2 provides an intuitive comparison between IV and SV means for the importance items.

```
# sort_by: "none", "min", "max", "mid", "hurwicz"
p2 <- plot_means(df_I, labels=LABELS, show=c("theta","H","
     sv"), sort_by="hurwicz", sort_data="theta", sort_order
     ="desc", gamma=0.5)
```

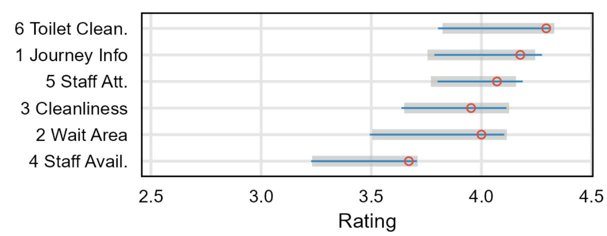


Fig. 2: Importance means: SV as red circles, θ-based as wide gray segments, H-based as thin blue segments.

E. Visual Descriptives

plot_distributions() visualizes response distributions across multiple items. For IV data, IAA visualizations and IVD summaries are supported. For SV data, RFH plots can be generated and overlaid with IAA when comparable. The following examples use Items I1–I3 for illustration. Fig. 3 shows IAA FSs overlaid on SV RFHs for comparing the overall distribution between IV and SV data.

```
# type: "iaa", "rfh", "iaa_rfh", "iaa_ivd"
p3 <- plot_distributions(iv_data, sv_data, items=Imp[1:3],
    type="iaa_rfh", ncol=3)
```

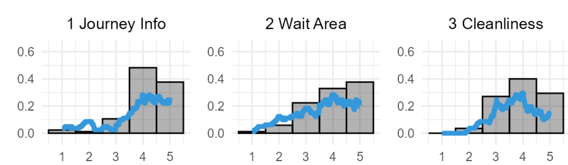
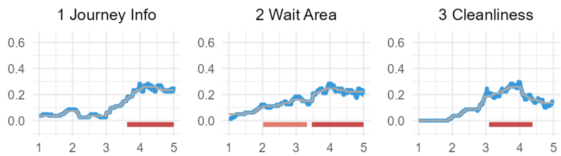


Fig. 3: IAA FSs overlaid on SV RFHs for Items I1–I3.

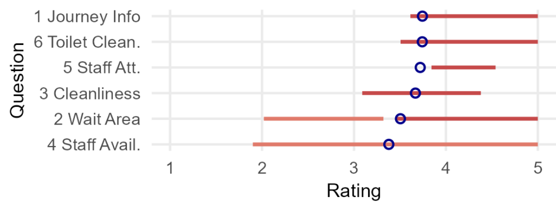
```
p4_1 <- plot_distributions(iv_data, items=Imp[1:3],
       type="iaa_ivd", ncol=3)
```

plot_ivd_summary() provides an item-wise summary view of IVD outputs. Items can be ranked by defuzzification of their corresponding IAA FSs. Color intensity reflects the peak height of each IAA FS. Fig. 4 shows IAA–IVD plots and the IVD summary for cross-item distribution comparison.

```
# ivd_method: "DUW" (default), "DU", "NUW", "NU", "IALC"
# sort_by: "none", "centroid", "mom", "alc", ...
p4_2 <- plot_ivd_summary(iv_data, items=Imp[1:6],
       labels=LABELS, ivd_method="DUW",
       sort_by="centroid", sort_order="desc")
```



(a) IAA FSs with IVD summaries for Items I1–I3.



(b) IVD summary across Items I1–I6.

Fig. 4: Examples of IAA and IVD visualizations.

F. Psychometric Assessment

calc_cor() computes correlation coefficients. The `method` parameter specifies the correlation method. While not demonstrated in this study, the H-based correlation also accommodates discontinuous intervals [23].

```
# method: "theta" (default), "hausdorff", "midpoint", "
    arithmetic"
> calc_cor(df_I$theta, df_I$sv, method="theta")    # 0.968
```

Here, the correlation coefficients between IV and SV means are calculated to assess convergent validity of the IV response format. Since this is a between-subject study, correlation is computed at the group level via SV and IV means.

plot_correlation() visualizes the correlation, accepting multiple x against a single y benchmark. When multiple x variables are provided, the `method` parameter can specify a corresponding method for each, as shown in Fig. 5.

```
p5 <- plot_correlation(x=list(theta=df_I$theta, H=df_I$H),
    y=df_I$sv, method=c("theta", "hausdorff"))
```

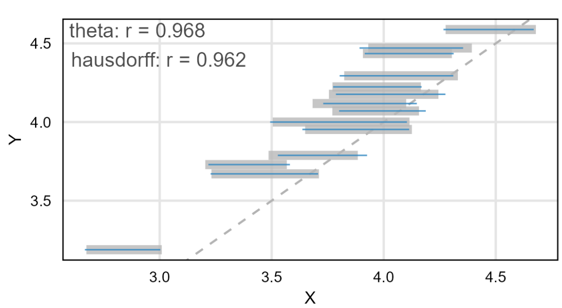


Fig. 5: Correlation between IV and SV means for I1-I14.

calc_reliability() computes Cronbach's α for IV and SV item sets. The function accepts one or more data sources and a list of item groups, computing α for all combinations.

```
# method: "theta" (default), "hausdorff"
> calc_reliability(iv_data, sv_data, items=list(S=Sat, I=
    Imp), method="theta")
#      data items method  alpha n_items
# 1 iv_data     S  theta 0.8868      14
# 2 iv_data     I  theta 0.8567      14
# 3 sv_data     S     sv 0.8995      14
# 4 sv_data     I     sv 0.8844      14
```

G. Diagnostic Analysis

calc_gap() computes item-wise gaps as perception minus expectation ($P - E$). For IV data, interval subtraction is applied.

```
> calc_gap(expectation=df_I$theta, perception=df_S$theta)
#   Aspect Perception_L ... Expectation_L ... Gap_L  Gap_U
```

plot_gap() visualizes expectation–perception gaps. In Fig. 6, expectation intervals are thick/light segments and perception intervals are thin/dark segments. SV points (red circles) are overlaid for comparison.

```
p6 <- plot_gap(labels=LABELS,
      perception=df_S$theta, expectation=df_I$theta,
      sv_perception=df_S$sv, sv_expectation=df_I$sv)
```

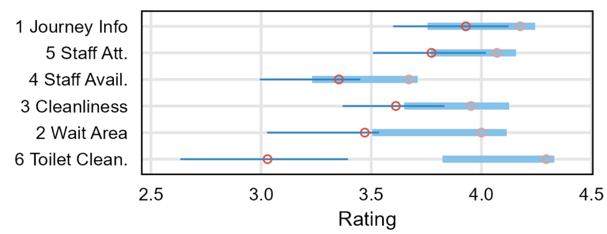


Fig. 6: Gap analysis with IV and SV representations.

plot_ipa() generates IPA visualizations. For IV inputs, quadrant boundaries are shaded regions rather than single lines. Fig. 7 illustrates SV-based and IV-based IPA.

```
p7_1 <- plot_ipa(df_I$sv, df_S$sv)
p7_2 <- plot_ipa(df_I$theta, df_S$theta)
```

calc_csi() computes CSI via importance-weighted aggregation. CSI is numeric for SV data and an interval for IV data.

```
> calc_csi(df_I$sv, df_S$sv)       # 3.511
> calc_csi(df_I$theta, df_S$theta) # [3.127, 3.659]
```

H. Data Export

save_results() exports tabular outputs to Excel for reporting and further analysis. IV data stored as nested lists are unpacked into separate columns.

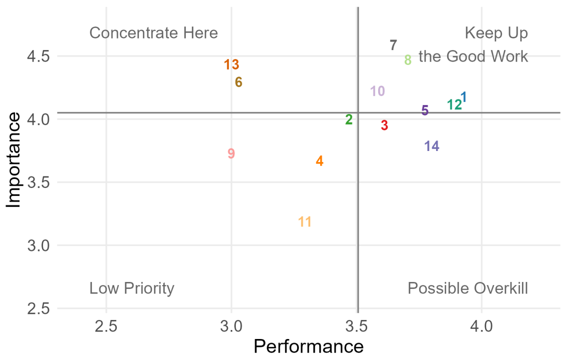
```
save_results(df_I, file="Descriptives", dir="output/")
```

save_plot() saves figures with consistent defaults. Plots are `ggplot2` objects, allowing further customization.

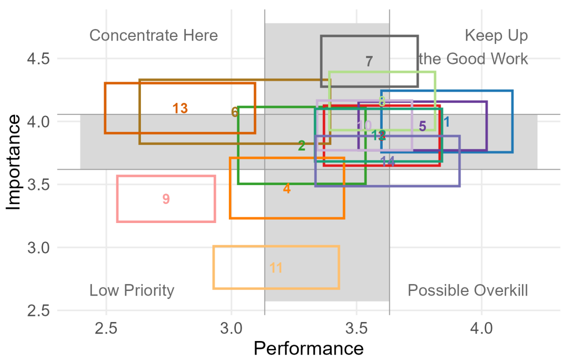
```
save_plot(p7_2, "IPA_IV", dir="output/")
```

IV. Conclusion and Future Work

This paper introduces `ISurvey`, an R toolkit addressing the limited availability of accessible software for IV survey data analysis. `ISurvey` provides methods such as IAA, IVD, Fréchet-based descriptive statistics under d_θ and d_H, correlation measures, and Cronbach's α, as well as IV extensions of gap analysis, IPA, and CSI. Through a rail customer case study, this paper demonstrated a complete workflow—from DECSYS export through analysis to reporting—comparing



(a) SV-based IPA.



(b) IV-based IPA with IV quadrant boundaries.

Fig. 7: IPA comparison using SV and IV data.

IV and SV responses to illustrate how the toolkit supports researchers in evaluating whether IV data provide additional insights compared to traditional SV methods.

Future work will focus on three directions. First, the toolkit will be extended to incorporate additional methods, including further developments in IV statistics, ranking and multi-criteria decision-making methods, and additional diagnostic methods. Second, although H-based Cronbach's α is implemented following recent work on H-based means and variances, its mathematical properties have not yet been fully established. We will investigate its theoretical properties and empirical behavior, particularly in comparison with θ-based Cronbach's α. Third, while this paper focuses on software implementation, future studies will examine the motivations for IV extensions of diagnostic methods such as gap analysis, IPA, and CSI, and their value in applied research, including what additional insights IV data may offer and how resulting decisions compare with those from traditional SV methods.

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