Fuzzy Reliability Redundancy Allocation Problem Using Immigrants-based Multifactorial Evolutionary Algorithm

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Abstract— **High system reliability is essential for failure-free operation of engineering systems. The Reliability Redundancy Allocation Problem (RRAP) aims to maximize system reliability through optimal component selection and redundancy allocation under resource constraints. As an NP-hard problem, RRAP is typically solved using metaheuristics, which often suffer from premature convergence, especially in constrained environments. Additionally, uncertainty in system parameters motivates the use of fuzzy RRAP (FRRAP) formulations. To address these challenges, this paper proposes an immigrant-based multifactorial evolutionary algorithm (I-MFEA) with elitist, random, and hybrid immigrant strategies for solving FRRAPs. The approach incorporates a penalty-based constraint-handling mechanism and is evaluated on multitasking fuzzy RRAP benchmarks, which includes series and complex bridge systems. Taguchi method is used for the parameter tuning. The performance is assessed using average, best, and standard deviation of reliability, and compared with baseline MFEA. Further, TOPSIS based ranking method is utilized to rank the approaches based on their achieved reliability. The experimental results demonstrate that the elitist I-MFEA consistently achieves superior performance, highlighting the effectiveness of the proposed framework.**

Keywords—EMTO, MFEA, Immigrants, Fuzzy sets, System Reliability Optimization, Constrained Optimization, Fuzzy Series System, Fuzzy Complex Bridge System.

I. Introduction

In reliability engineering, system reliability represents the probability of failure-free operation over a specified period under given conditions (e.g., cold, warm, or hot) [1]. High reliability is critical for ensuring un-interrupted service in many sectors such as transportation, communications, and power systems, as well as in applications like pharmaceutical plants and safety-critical systems [2], [3].

System reliability can be improved through two approaches: (a) reliability optimization by enhancing component reliability, and (b) redundancy allocation (RAP) by adding redundant components [4]. While the former increases cost, the latter raises cost, weight, and volume. Their joint consideration leads to the reliability redundancy allocation problem (RRAP) [5], [6] which is challenging due to mixed discrete-continuous variables and nonlinear system behavior. Consequently, RRAP is classified as an NP-hard problem [7], limiting the effectiveness of classical methods such as dynamic programming and branch-and-bound [8], [9]. To address this, metaheuristic approaches have been widely adopted [10] - [12]. RRAP has been studied as single-, multi-, and many-objective optimization problems, which enables trade-off analysis among reliability and resource constraints [13]-[15]. Recently, evolutionary multitask optimization (EMTO) has been applied to solve multiple RRAPs simultaneously [16]-[18]. In practical scenarios, system parameters such as reliability, cost, volume and weight are often uncertain due to design variability and environmental factors [14]. Fuzzy modeling has therefore been incorporated into RRAP (FRRAP) to better capture uncertainty. While most studies address FRRAP independently, limited work has explored multitasking frameworks, such as the MFEA-based approach in [17] and its multi-objective extension in [19]. However, EMTO methods are still prone to premature convergence, particularly in constrained problems like RRAP. Introducing immigrant strategies is a promising approach to enhance diversity and avoid local optima [20], [21], [22].

Motivated by this, we propose an immigrant-based multi-factorial evolutionary algorithm (I-MFEA) for solving multitasking FRRAPs. Three variants: elitist, random, and hybrid are developed and integrated with a penalty-based constraint-handling mechanism. The proposed approaches are evaluated on fuzzy series and complex bridge systems. Taguchi method is used for parameter tuning and performance is compared with the baseline MFEA under identical settings.

The remainder of the paper is organized as follows: Section II presents the Fuzzy RRAP formulation, Section III describes the proposed methodology, Section IV details the experimental setup and results, and Section V concludes with limitations and future work.

II. Problem Formulation

This section presents the considered fuzzy RRAPs, integrates constraint handling into the reliability formulation and models them as distinct tasks within the proposed I-MFEA framework. We adopt benchmark problems from Kuo and Prasad [2] and fuzzy formulation from [17]. The symbols and their definitions are listed in Table I. The general formulation of RRAP can be written as follows:

$$Maximize\ F = R_s(r, n) \quad (1)$$
$$subject\ to,$$
$$g_j(r, n) \le b_j; j = 1,2,3 \dots \dots \dots k$$

Here, R_s denotes overall system reliability based on component reliabilities (r) and redundancy levels (n). K is the total

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Table I. Symbol and their definition.

Symbol	Definition	Symbol	Definition
$\boldsymbol{R_S(r,n)}$	Overall reliability of the system.	r_i	i^{th} sub-system's component reliability.
$\boldsymbol{Q_i}$	i^{th} sub-system's failure probability	n_i	i^{th} sub-system's redundant components.
$\boldsymbol{R_i}$	i^{th} sub-system's reliability.	$v_{i,}$	i^{th} sub-system's volume
$\boldsymbol{V}$	Maximum limit for volume constraint	$w_{i,}$	i^{th} sub-system's weight
$\boldsymbol{C}$	Maximum limit for cost constraint	α_i	i^{th} sub-system's shaping factor
$\boldsymbol{W}$	Maximum limit for weight constraint	β_i	i^{th} sub-system's scaling factor
$\boldsymbol{k}$	Total constraints	g_j	j^{th} resource constraint
$\boldsymbol{m}$	Number of sub-systems.	b_j	j^{th} constraint's upper limit.

number of constraints with g_j and b_j representing the j^{th} constraint and it's upper bound. For a system of m series-connected subsystems, the reliability function in Eq. (1) is formulated as follows.:

$$R_s(r,n) = \prod_{i=1}^{m} R_i(r_i,n_i) = \prod_{i=1}^{m}[1 - Q_i^{n_i}] = \prod_{i=1}^{m}[1-(1-r_i)^{n_i}] \quad (2)$$

Here, (i) is representing i^{th} subsystem. R_i is the reliability for i^{th} sub-system, which is based on i^{th} sub-systems components reliability (r_i) and the number of redundant components (n_i), respectively. The calculation of R_i is based on the complements of the i^{th} sub-system failure probability Q_i. In this work, two popular reliability optimization problems are investigated, namely, a series system and a complex (bridge) system. Their mathematical formulations are presented below:

A. Series System

It consists five sequential subsystems (Fig. 1(a)), and the non-linear RRAP formulation can be written as follows:

$$Maximize\ R_S(r,n) = \prod_{i=1}^{m}\ [1-(1-r_i)^{n_i}] \quad (3)$$

subject to,

$$g_1(n) = \sum_{i=1}^{m} v_i n_i^2 - \tilde{V} \leq 0$$

$$g_2(r,n) = \sum_{i=1}^{m} \alpha_i\left(-\frac{1000}{\ln(r_i)}\right)^{\beta_i}\left[n_i + \exp\left(\frac{n_i}{4}\right)\right] - \tilde{C} \leq 0$$

$$g_3(n) = \sum_{i=1}^{m} w_i * n_i * \exp(0.25n_i) - \widetilde{W} \leq 0$$

$$0.5 \leq r_i \leq 1, r_i \in [0,1] \subset \mathbb{R}^+;\ 1 \leq n_i \leq 5, n_i \in Z^+; i = 1,2,\dots m.$$

B. Complex Bridge System

It also consists five subsystems (Fig. 1(b)), and the non-linear RRAP formulation can be written as follows:

$$Maximize\ R_s(r,n) = R_1R_2 + R_3R_4 + R_1R_4R_5 + R_2R_3R_5 - R_1R_2R_3R_4 - R_1R_2R_3R_5 - R_1R_2R_4R_5 - R_1R_3R_4R_5 - R_2R_3R_4R_5 + 2R_1R_2R_3R_4R_5 \quad (4)$$

Subject to,

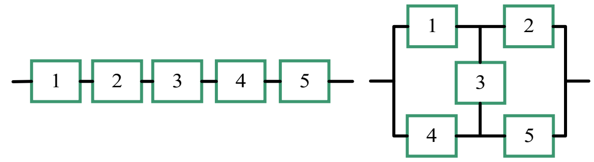
$$g_1(n) = \sum_{i=1}^{m} v_i n_i^2 - \tilde{V} \leq 0$$

$$g_2(r,n) = \sum_{i=1}^{m} \alpha_i\left(-\frac{1000}{\ln(r_i)}\right)^{\beta_i}\left[n_i + \exp\left(\frac{n_i}{4}\right)\right] - \tilde{C} \leq 0$$

$$g_3(n) = \sum_{i=1}^{m} w_i * n_i * \exp(0.25n_i) - \widetilde{W} \leq 0$$

$$0.5 \leq r_i \leq 1, r_i \in [0,1] \subset \mathbb{R}^+;\ 1 \leq n_i \leq 5, n_i \in Z^+; i = 1,2,\dots m$$

Here, $R_s(r,n)$ denotes the system reliability to be maximized for a system with $m = 5$ subsystems, where $r_i \in [0,1]$ and $n_i \in \{1,\dots,5\}$, resource constraints g_1, g_2, g_3 represents



(a) Series System (b) Complex Bridge System

Fig. 1. System structure of both the case studies.

volume, cost, and weight, respectively, and their fuzzified upper bounds $(\widetilde{W}, \tilde{V}, \tilde{C})$ are modeled using Type-1 fuzzy sets with triangular membership functions defined over [0,1], which can be expressed as follows:

$$\tilde{A} = \{(x, \mu_A(x)) | \forall_x \in X\} \quad (5)$$

For the defuzzification, the popular centroid defuzzification is used. "w_i" and "v_i" indicates the i^{th} sub-systems weight and volume, respectively. Further, "α_i" and "β_i" indicates i^{th} sub systems shaping and scaling factors, respectively.

C. Constraint Handling Technique

As discussed, RRAP is a constrained optimization problem, which requires effective constraint handling. In this study, a penalty-based approach [11], [23] is employed, where constraint violations are incorporated into the objective function proportional to their magnitude. Accordingly, the system reliability objective is reformulated as follows:

$$R_{total}(x) = \begin{cases} -R_s(x) + \psi(x), & if\ g_y(x) > b_y \\ -R_s(x), & otherwise \end{cases} \quad (6)$$

Where, $R_s(x)$ denotes the overall system reliability, $g_y(x)$ represents the y^{th} resource constraint, b_y is its upper bound, and $\psi(x)$ is the total penalty applied when constraints are violated. The penalty term is defined as:

$$\psi(x) = \sum_{y=1}^{M} \phi_y * \left(\max\left(0, g_y(x) - b_y\right)\right)^2 \quad (7)$$

Here, M denotes the number of constraints and ϕ_y is the penalty coefficient for the y^{th} constraint, with all constraints treated equally by setting $\phi_y = 1$. The decision vector $x = (r,n)$ includes component reliabilities and redundancy levels. For computational convenience, the problem is formulated as a minimization problem, and the final system reliability R_s is obtained by removing the negative sign after optimization.

D. Multi-Task Settings of Fuzzy-RRAPs

As an EMTO-based method, the proposed I-MFEA-FRRAP simultaneously solves two fuzzy RRAPs treated as independent tasks: Task-1 (series) and Task-2 (bridge) collectively forming the test set.

III. Solution Approach

This section introduces the I-MFEA for multitasking fuzzy RRAPs, extending MFEA with immigrant strategies to enhance diversity and mitigate premature convergence. The following subsection outlines its core principles and working mechanism.

A. Immigrants-based MFEA for Fuzzy-RRAPs

The proposed immigrant-based multifactorial evolutionary algorithm (I-MFEA) extends the standard MFEA by

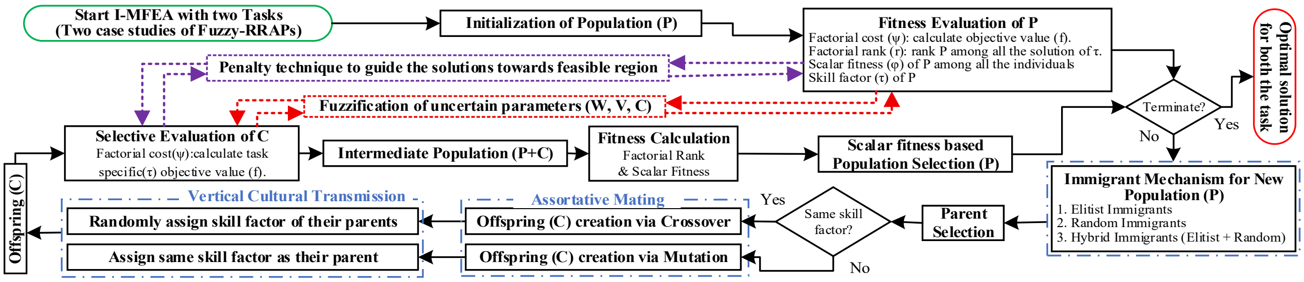


Fig. 2. Working of I-MFEA-FRRAPs.

incorporating immigrant-based strategies, while retaining its core framework (see Gupta et al. [24] for details). The approach is characterized by: (a) a unified search space across tasks, (b) implicit knowledge transfer, and (c) the integration of immigrant solutions during evolution.

The key advantage of I-MFEA lies in its multitasking capability, where simpler tasks assist more complex ones through knowledge transfer, leading to faster convergence compared to independent optimization. Additionally, immigrant strategies: elitist, random, and hybrid enhance diversity and help escape local optima, improving the likelihood of global convergence. Knowledge transfer is regulated by the random mating probability (rmp) [24], which controls cross-task mating and information exchange. While rmp accelerates convergence, the immigrant mechanism maintains diversity. In particular, elitist immigrants introduce high-quality solutions into the population. Accordingly, three variants are proposed: I-MFEA Elitist, I-MFEA Random, and I-MFEA Hybrid. The detailed working of the proposed I-MFEA for fuzzy RRAPs is presented in the following subsection.

B. Working of I-MFEA for Solving Multiple Fuzzy RRAPs

This subsection describes the procedure for solving multiple fuzzy RRAPs using the proposed I-MFEA. The multitasking test set (Section II) includes two problems: a fuzzy series system (Task-1) and a fuzzy bridge system (Task-2), which are optimized simultaneously. As illustrated in Fig. 2, the I-MFEA workflow consists of five stages: (1) population initialization, (2) fitness evaluation and population selection, (3) immigrant mechanism and population reformulation, (4) offspring generation, and (5) termination criteria.

1) Stage-1: Population Initialization

The population P is initialized by generating unified chromosomes y_i based on the maximum task dimensionality. In this case, both tasks have equal dimensionality (10), so all chromosomes are represented as fixed-length real-valued vectors in [0,1]. A representative structure is shown in Fig. 3.
Each chromosome maintains four attributes: factorial cost, factorial rank, scalar fitness, and skill factor (see [24] for details). To obtain task-specific solutions, chromosomes are decoded using a continuous mapping scheme. Half of the variables correspond to component reliabilities (r_i), while the remaining represent redundancy levels (n_i), with the latter obtained via rounding. These decoding maps unified chromosomes to task-specific decision variables according to the problem structure and constraints.

2) Stage-2: Fitness Evaluation and Selection of New Population

Before fitness evaluation, task-specific decoding maps each unified solution to the decision variables of its associated task based on its skill factor. For example, when the skill factor is 1, the solution is decoded for Task-1 (series system), which has 10 variables equally divided into component reliabilities r_i and redundancy levels n_i. A continuous mapping scheme [24] is used, where decoded values define reliabilities directly, and redundancy levels are obtained by rounding to the nearest integer. This decoding strategy is consistently applied across all fuzzy RRAP case studies.

Unified Search Space / Chromosome									
0.5510	0.7428	0.7818	0.7828	0.7258	0.4114	0.0327	0.2446	0.0551	0.6730
d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}

Fig. 3. A sample unified search space

The proposed I-MFEA employs two types of fitness evaluation: initial and selective. In the initial evaluation, each individual is assessed on all tasks to determine its skill factor. Factorial costs (penalized system reliability values) are computed using the constraint-handling method described in subsection II-C, followed by the calculation of factorial ranks and scalar fitness for cross-task comparison. In selective evaluation, individuals are evaluated only on their assigned task, reducing computational cost while maintaining task-specific performance assessment. Constraint handling is again applied using the penalty-based approach.

Finally, population selection is based on scalar fitness, where higher-quality individuals are retained, which ensures elitism and preserves superior solutions across generations.

3) Stage-3: Immigrant Mechanism and New Reformed-Population (P')

This stage focuses on introducing immigrant solutions *(IS)* into the evolutionary process to enhance population diversity and prevent premature convergence. The immigrant solutions generated through the proposed immigrant mechanism are combined with the existing population and an equal number of the worst-performing individuals are replaced to form the reformed population P'. The following steps explains the overall procedure describing how the considered immigrant mechanisms: elitist, random, and hybrid interact with the existing population P to produce the reformed population P'.

a) Stage 3.1: Selection of Parents of Immigrants (PI) and Generation of Candidate Immigrants (CI):

Immigrant parents PI are selected based on the predefined number NI and strategy: elitist (top NI from population P), random (NI randomly generated individuals), or hybrid (half elitist, half random). The resulting PI ensures coverage across all tasks. Candidate immigrants CI are then produced via mutation and inherit the skill-factor of their respective parents for task-specific evaluation.

b) Stage-3.2: Selection of Immigrant Solutions (IS) and Formation of the Reformed Population (P'):

Each candidate immigrant solution is evaluated via factorial cost, and is accepted as an immigrant solution *(IS)* if it improves upon its parent; otherwise, it is discarded. The accepted *IS* replace an equal number of worst individuals in population P, forming the reformed population P', which is then used in subsequent evolutionary operations.

4) Generating Offspring (C) from the Reformed Population (P')

New offspring are generated by selecting parent individuals from the reformed population and applying crossover or mutation. Specifically, two parents P_a and P_b are randomly selected. If they share the same skill factor or satisfy the random mating probability (rmp), simulated binary crossover (SBX) is applied (assortative mating) to produce offspring. Otherwise, mutation is used. Offspring inherit skill factors through vertical cultural transmission: for crossover, each offspring randomly inherits a parent's skill factor, while for mutation, each offspring inherits from its respective parent. Based on the assigned skill factor, offspring undergo selective evaluation as described in *Stage 2*. The offspring are then merged with the parent population to form an intermediate population. Fitness values are recalculated to obtain factorial ranks and scalar fitness, and elitist selection is applied to form the next generation, it ensures the survival of high-quality solutions.

5) Termination Criteria

The process iterates from *Stage-2* until termination (maximum iterations). The final output provides near-optimal solutions for both fuzzy RRAPs.

IV. Experimental Results and Comparison Analysis

For experiments, two fuzzy RRAP case studies (Section II) are evaluated in MATLAB 2024a on a 24 GB RAM Intel Core i5 (2.00 GHz) system, with input data in Table II including scaling/shaping factors, component volume/weight, and fuzzified upper bounds of volume, cost, and weight.

Table II. Input data for both the tasks.

i	$10^5\alpha_i$	β_i	v_i	w_i	$\widetilde{V}$	$\widetilde{C}$	$\widetilde{W}$
1	2.330	1.5	1	7			
2	1.450	1.5	2	8			
3	0.541	1.5	3	8	111.11	181.6	204.37
4	8.050	1.5	4	6			
5	1.950	1.5	2	9			

Table III. L-9 Orthogonal setting for Taguchi based design of experiments.

Experiments	parameters			
	population	mu	rmp	immigrants * 100 (%)
Exp-1	50	5	0.3	0.05
Exp-2	50	10	0.6	0.1
Exp-3	50	15	0.8	0.15
Exp-4	100	5	0.6	0.15
Exp-5	100	10	0.8	0.05
Exp-6	100	15	0.3	0.1
Exp-7	150	5	0.8	0.1
Exp-8	150	10	0.3	0.15
Exp-9	150	15	0.6	0.05

A. Parameter Tuning Based on Taguchi Assisted L-9 Orthogonal Array

Parameter tuning is performed using the Taguchi design of experiments [25], which identifies optimal parameter settings with minimal experiments via orthogonal arrays. For MFEA, the parameters include population size, crossover index (mu), and random mating probability (rmp), while I-MFEA introduces an additional parameter, the immigrant ratio. The parameter levels are: population [50, 100, 150], mu [5, 10, 15], rmp [0.3, 0.6, 0.8], and immigrants [5%, 10%, 15%]. Based on these, an L9 orthogonal array is employed (Table III). For parameter tuning, five independent runs are conducted for each setting, and average system reliability is recorded for both tasks (Table IV). To address the multitasking, signal-to-noise ratio (SNR) analysis is applied, and a combined average SNR is computed. The configuration with the lowest average SNR is selected as optimal (Fig. 4), with best settings highlighted. These optimal configurations are then used for 20 independent runs in the final performance evaluation.

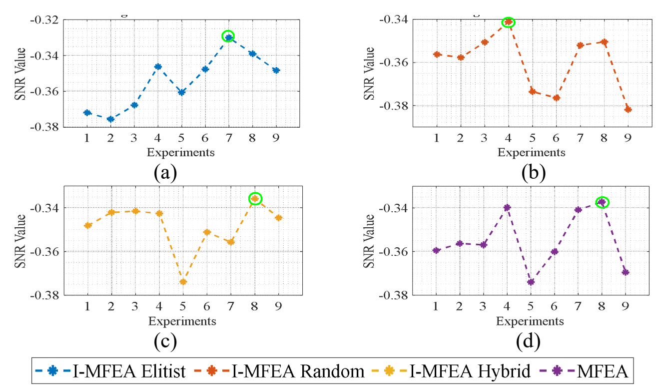
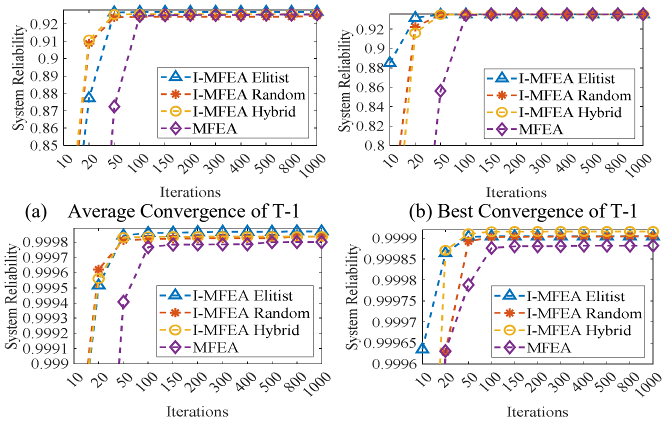


Fig. 4. Taguchi experiment results showing average SNR values for the FRRAP test set using (a) I-MFEA Elitist, (b) I-MFEA Random, (c) I-MFEA Hybrid, and (d) MFEA.

B. Experimental Results of Fuzzy RRAPs (T-1:Fuzzy Series System and T-2: Fuzzy Complex (Bridge) System)

This section presents a comparative analysis of the proposed I-MFEA variants and MFEA in terms of average and best convergence, as well as final solution quality for two test problems: T-1 (fuzzy series system) and T-2 (fuzzy complex bridge system). Tables V and VI report the average system reliability (with standard deviation) and best reliability values across iterations (10-1000) for all approaches. The corresponding convergence behavior is illustrated in Fig. 5. For T-1, Fig. 5(a) and (b) show that all I-MFEA variants (Elitist, Random, and Hybrid) converge faster than MFEA.



(a) Average Convergence of T-1 (b) Best Convergence of T-1

(c) Average Convergence of T-2 (d) Best Convergence of T-2

Fig. 5. Convergence comparison of I-MFEA variants and MFEA: (a) average and (b) best reliability for T-1, and (c) average and (d) best reliability for T-2 over iterations.

Specifically, the proposed variants achieve stability within ~50 iterations for both average and best convergence, whereas MFEA requires approximately 100 iterations.

Fig. 5(c) and (d) present the average and best convergence for T-2. Similar to T-1, all I-MFEA variants converge faster, stabilizing around 50 iterations, whereas MFEA requires approximately 100 iterations for both average and best convergence.

Table V shows that I-MFEA Elitist achieves the highest average reliability for both tasks. However, the best reliability is obtained by I-MFEA Random for T-1 and I-MFEA Hybrid for T-2 (Table VI). Table VII summarizes the optimal solutions, including system reliability, decision variables (component reliabilities and redundancy levels), and resource constraints. All approaches yield feasible solutions. For T-1, I-MFEA Random achieved the highest reliability, while for T-2, I-MFEA Hybrid outperforms the other variants and MFEA.

Table IV. Optimal parameter settings of I-MFEA variants and MFEA for test set identified using the lowest average SNR values of both tasks.

Exp.	I-MFEA Elitist					I-MFEA Random					I-MFEA Hybrid					MFEA				
	T-1	T-2	SNR_T1	SNR_T2	SNR	T-1	T-2	SNR_T1	SNR_T2	SNR	T-1	T-2	SNR_T1	SNR_T2	SNR	T-1	T-2	SNR_T1	SNR_T2	SNR
Exp-1	0.91816	0.99975	-0.37084	-0.00108	-0.37191	0.92148	0.99976	-0.35515	-0.00106	-0.35621	0.92317	0.99979	-0.34716	-0.00093	-0.34809	0.92076	0.99977	-0.35851	-0.00099	-0.35951
Exp-2	0.91734	0.99981	-0.37472	-0.00084	-0.37556	0.92115	0.99978	-0.35670	-0.00098	-0.35768	0.92447	0.99978	-0.34109	-0.00096	-0.34205	0.92149	0.99972	-0.35512	-0.00120	-0.35631
Exp-3	0.91901	0.99979	-0.36679	-0.00093	-0.36772	0.92263	0.99980	-0.34972	-0.00087	-0.35060	0.92459	0.99976	-0.34049	-0.00103	-0.34152	0.92128	0.99979	-0.35606	-0.00092	-0.35698
Exp-4	0.92356	0.99980	-0.34533	-0.00087	-0.34620	0.92461	0.99984	-0.34039	-0.00071	**-0.34110**	0.92431	0.99982	-0.34180	-0.00076	-0.34257	0.92493	0.99980	-0.33890	-0.00089	-0.33978
Exp-5	0.92052	0.99978	-0.35967	-0.00094	-0.36061	0.91779	0.99978	-0.37258	-0.00097	-0.37356	0.91770	0.99981	-0.37298	-0.00084	-0.37383	0.91770	0.99975	-0.37298	-0.00109	-0.37407
Exp-6	0.92326	0.99979	-0.34677	-0.00090	-0.34768	0.91716	0.99979	-0.37554	-0.00093	-0.37646	0.92251	0.99980	-0.35028	-0.00085	-0.35113	0.92061	0.99979	-0.35926	-0.00089	-0.36015
Exp-7	0.92695	0.99987	-0.32943	-0.00056	**-0.32999**	0.92231	0.99982	-0.35123	-0.00078	-0.35201	0.92153	0.99982	-0.35491	-0.00080	-0.35571	0.92465	0.99985	-0.34025	-0.00064	-0.34089
Exp-8	0.92506	0.99982	-0.33828	-0.00078	-0.33906	0.92267	0.99980	-0.34955	-0.00088	-0.35043	0.92575	0.99984	-0.33507	-0.00071	**-0.33578**	0.92548	0.99980	-0.33633	-0.00087	**-0.33720**
Exp-9	0.92311	0.99980	-0.34748	-0.00086	-0.34833	0.91606	0.99975	-0.38077	-0.00110	-0.38187	0.92392	0.99980	-0.34364	-0.00087	-0.34452	0.91864	0.99977	-0.36857	-0.00098	-0.36955

Table V. Iteration-wise average and standard deviation of reliability for both the tasks

Average Convergence	Iterations	10	20	50	100	150	200	300	400	500	800	1000
T-1 (Series System)	I-MFEA Elitist	0.750343 ±1.05E-01	0.877250 ±4.08E-02	0.926575 ±7.51E-03	0.926903 ±7.55E-03	0.926927 ±7.57E-03	0.926934 ±7.57E-03	0.926943 ±7.57E-03	0.926946 ±7.57E-03	0.926947 ±7.57E-03	0.926949 ±7.57E-03	**0.926951** ±7.57E-03
	I-MFEA Random	0.765753 ±1.11E-01	0.908743 ±2.25E-02	0.923851 ±8.37E-03	0.924101 ±8.40E-03	0.924139 ±8.39E-03	0.924162 ±8.39E-03	0.924181 ±8.39E-03	0.924184 ±8.39E-03	0.924191 ±8.38E-03	0.924200 ±8.38E-03	0.924615 ±8.51E-03
	I-MFEA Hybrid	0.779352 ±7.66E-02	0.910747 ±1.20E-02	0.925440 ±7.83E-03	0.925665 ±7.79E-03	0.925707 ±7.79E-03	0.925722 ±7.79E-03	0.925728 ±7.80E-03	0.925737 ±7.79E-03	0.925742 ±7.79E-03	0.925746 ±7.80E-03	0.925749 ±7.80E-03
	MFEA	0.592591 ±1.67E-01	0.686141 ±1.42E-01	0.872376 ±5.70E-02	0.924566 ±9.85E-03	0.925071 ±9.73E-03	0.925131 ±9.72E-03	0.925175 ±9.71E-03	0.925196 ±9.72E-03	0.925411 ±9.57E-03	0.925473 ±9.56E-03	0.925480 ±9.56E-03
T-2 (Bridge System)	I-MFEA Elitist	0.998126 ±1.77E-03	0.999515 ±4.19E-04	0.999843 ±3.75E-05	0.999861 ±3.12E-05	0.999861 ±3.12E-05	0.999865 ±2.36E-05	0.999867 ±2.51E-05	0.999867 ±2.51E-05	0.999867 ±2.51E-05	0.999871 ±2.72E-05	**0.999871** ±2.72E-05
	I-MFEA Random	0.998180 ±3.19E-03	0.999620 ±2.64E-04	0.999812 ±6.37E-05	0.999819 ±6.40E-05	0.999823 ±6.56E-05	0.999823 ±6.56E-05	0.999826 ±6.68E-05	0.999826 ±6.68E-05	0.999826 ±6.68E-05	0.999836 ±5.99E-05	0.999836 ±5.99E-05
	I-MFEA Hybrid	0.998103 ±2.06E-03	0.999560 ±2.31E-04	0.999829 ±6.57E-05	0.999835 ±6.54E-05	0.999836 ±6.54E-05	0.999836 ±6.54E-05	0.999836 ±6.53E-05	0.999836 ±6.53E-05	0.999836 ±6.53E-05	0.999836 ±6.53E-05	0.999836 ±6.53E-05
	MFEA	0.992014 ±1.32E-02	0.997072 ±4.94E-03	0.999407 ±5.90E-04	0.999767 ±9.01E-05	0.999783 ±8.98E-05	0.999785 ±9.01E-05	0.999785 ±9.02E-05	0.999785 ±9.02E-05	0.999800 ±6.70E-05	0.999800 ±6.72E-05	0.999800 ±6.72E-05

Table VI. Best reliability across iterations for both the tasks

Best Convergence	Iterations	10	20	50	100	150	200	300	400	500	800	1000
T-1 (Series System)	I-MFEA Elitist	0.885196	0.931755	0.935190	0.935337	0.935357	0.935358	0.935372	0.935375	0.935375	0.935389	0.935389
	I-MFEA Random	0.721268	0.922381	0.934976	0.935386	0.935388	0.935388	0.935388	0.935388	0.935388	0.935388	**0.935392**
	I-MFEA Hybrid	0.648727	0.916134	0.934989	0.935210	0.935328	0.935328	0.935353	0.935353	0.935353	0.935353	0.935355
	MFEA	0.472438	0.614285	0.856322	0.934542	0.935240	0.935331	0.935365	0.935375	0.935379	0.935391	0.935391
T-2 (Bridge System)	I-MFEA Elitist	0.999635	0.999864	0.999903	0.999904	0.999904	0.999904	0.999904	0.999904	0.999904	0.999904	0.999904
	I-MFEA Random	0.998369	0.999628	0.999892	0.999902	0.999903	0.999903	0.999903	0.999903	0.999904	0.999904	0.999904
	I-MFEA Hybrid	0.998732	0.999870	0.999910	0.999915	0.999915	0.999915	0.999915	0.999915	0.999916	0.999916	**0.999916**
	MFEA	0.997649	0.999631	0.999788	0.999876	0.999880	0.999881	0.999881	0.999882	0.999882	0.999882	0.999882

Table VII. Comparison of I-MFEA vs I-MFEA Random vs I-MFEA Hybrid vs MFEA in terms of best solutions obtained for both the tasks.

Best Solution	Approach	Component reliability (r_i)	Redundant Component (n_i)	System Reliability (R_s)	Volume (V)	Weight (W)	Cost (C)
T-1 (Series System)	I-MFEA Elitist	0.7839, 0.8755, 0.9057, 0.7161, 0.7927	[3, 2, 2, 3, 3]	0.935389	83.00	192.48	181.60
	I-MFEA Random	0.7839, 0.8749, 0.9053, 0.7172, 0.7924	[3, 2, 2, 3, 3]	0.935392	83.00	192.48	181.60
	I-MFEA Hybrid	0.7847, 0.8729, 0.9070, 0.7187, 0.7900	[3, 2, 2, 3, 3]	0.935355	83.00	192.48	181.60
	MFEA	0.7845, 0.8752, 0.9056, 0.7165, 0.7921	[3, 2, 2, 3, 3]	0.935391	83.00	192.48	181.60
T-2 (Bridge System)	I-MFEA Elitist	0.8322, 0.8611, 0.9158, 0.6552, 0.7080	[2, 3, 2, 3, 1]	0.999904	105.00	198.44	181.60
	I-MFEA Random	0.8319, 0.8616, 0.9164, 0.6534, 0.7139	[2, 3, 2, 4, 1]	0.999904	105.00	198.44	181.60
	I-MFEA Hybrid	0.7966, 0.8850, 0.8953, 0.6898, 0.6956	[3, 3, 2, 3, 1]	0.999916	84.00	202.96	181.60
	MFEA	0.8266, 0.8784, 0.9131, 0.6792, 0.6738	[2, 2, 2, 3, 1]	0.999882	83.00	189.43	181.60

Table VIII. Performance score and rank of the approaches.

Approach	S_i^+	S_i^-	P_i	Rank
I-MFEA Elitist	0.000000	0.000631	1.000000	1
I-MFEA Random	0.000631	0.000009	0.013855	4
I-MFEA Hybrid	0.000325	0.000307	0.485663	2
MFEA	0.000398	0.000234	0.369940	3

C. Ranking of the Algorithms

To rank the approaches, the TOPSIS multi-criteria decision-making method is employed [26]. The average system reliability values for both case studies are used as input criteria. Table VIII presents the resulting performance scores and rankings.

The results show that I-MFEA Elitist achieves the highest score and ranks first, followed by I-MFEA Hybrid and the baseline MFEA. The I-MFEA Random variant ranks last, indicating comparatively lower performance. Overall, the analysis confirms the superior effectiveness of the I-MFEA Elitist approach.

V. Conclusion and Future Work

This study proposes a novel evolutionary multitasking framework, termed as immigrant-based multi-factorial evolutionary algorithm (I-MFEA), for solving multiple fuzzy RRAPs. Three variants: Elitist, Random, and Hybrid are developed based on different immigrant strategies. The approach integrates a penalty-based constraint-handling mechanism and is evaluated on two benchmark problems: a fuzzy series system and a complex bridge system. Parameter tuning is performed using the Taguchi method and performance is assessed using average reliability, best reliability, and standard deviation. Comparative analysis with the baseline MFEA, along with TOPSIS-based ranking shows that the I-MFEA Elitist variant achieves superior performance and demonstrating the effectiveness of the proposed framework.

In this work, fuzziness is limited to the upper bounds of resource constraints. However, uncertainty may also exist in component reliability estimation [23]. In future work, will focus on incorporating fuzzy component reliabilities and extending the framework to large-scale systems to further evaluate scalability and robustness.

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