# Emergence of Spiral Waves in Locally Connected Neural Networks

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***Abstract*—Spiral waves are a representative class of spatiotemporal activity patterns widely observed in biological nervous systems and are closely related to neural information propagation. In this work, we investigate the emergence of spiral waves in a locally connected two-dimensional neural network composed of Aihara chaotic neurons. By systematically varying key parameters, including the local connection range, refractory-related parameters, feedback strength, network size, and boundary conditions, we examine dynamical behavior of the network under the different conditions. Numerical simulations show that spiral wave formation depends sensitively on the interplay between local connectivity and network parameters, and only emerges within specific parameter regions. Changes in network structures and parameters can significantly alter the spatiotemporal evolution of activity patterns. Our results clarify how refractoriness interact with local connectivity to regulate the emergence and stability of spiral wave patterns in neural networks.**

***Index Terms*—Spiral waves, Formation and propagation conditions, Chaitic neural networks, Locally connectd, Neural dynamics**

## I. Introduction

Complex spatiotemporal dynamics are ubiquitous in biological systems, and traveling waves represent one of the most fundamental organizational patterns, including plane waves and spiral waves [1]–[3]. Recent studies have demonstrated that traveling waves are prevalent across multiple spatial and temporal scales of neural activity in the brain [4, 5], and are closely associated with global dynamical states [6, 7]. These waves play a crucial role in inter-regional information transmission, coordination, and integration [3, 8, 9], and are involved in a wide range of brain functions, including perception [10, 11], memory [12, 13], and motor control [14, 15]. Among various forms of traveling waves, spiral waves have attracted particular attention due to their distinctive topological structures. A spiral wave is typically organized around a phase singularity, where oscillation amplitudes are markedly reduced and wavefronts rotate around the core [16].

In nervous systems, spiral waves are regarded as dynamically organizing units of population activity, capable of modulating oscillatory structures across space and time, influencing information transmission efficiency [3, 17], and being linked to working memory processes [12]. Evidence indicates that sleep stage 2 of human non-rapid eye movement (NREM), sleep spindle oscillations can self-organize into spiral waves propagating across the cortex, which are significantly correlated with post-sleep memory consolidation performance [18]. Moreover, the stability of these spiral wave patterns decreases with aging. The spatiotemporal stability of spiral waves may reflect neural dynamical features underlying memory-related functions [19].

Biological nervous systems do not rely solely on static connectivity structures for information processing, but instead encode and maintain information through dynamically evolving spatiotemporal patterns. Understanding these dynamical mechanisms is therefore fundamental for constructing information-processing models that are closer to the brain. Studying complex spatiotemporal dynamical patterns such as spiral waves in nervous systems using network models has provided important insights into how memory maintenance, information propagation, and large-scale neural coordination are realized in the brain [20]–[22]. Advances in the study of neural dynamics can deepen our understanding of information processing in artificial neural networks and promote the development of artificial intelligence models [23, 24].

Previous studies have shown that spiral wave patterns can spontaneously emerge in neural networks with local connectivity structures [16, 25]. However, the generation and propagation of spiral waves strongly depend on network structural parameters, neuronal dynamical parameters, and boundary conditions, and their systematic influence mechanisms remain to be further clarified. Motivated by these considerations, we construct a locally connected neural network with two dimension and systematically investigate the effects of local connection range, refractory-related parameters, feedback parameters, network size, and boundary conditions on the generation of spiral waves. From a dynamical perspective, this study aims to deepen the understanding of memory-related neural dynamics in the brain.

## II. Models

### A. Neural network model

The nervous system of the brain is composed of tens of billions of neurons connected in a complex but structured manner. Considering the existence of chaotic phenomena in brain dynamics, the Aihara chaotic neuron model proposed based on electrophysiological experiments can effectively describe chaos in nervous systems [16, 25]–[28]. Therefore, neurons in the nervous system are modeled by using the Aihara chaotic neuron. The Aihara chaotic neural network is a single-layer interconnected neural network that considers both external inputs and the summed interactions with other neurons. The network dynamical equations are given as follows:

$$\boldsymbol{\eta}(t) = k_f \boldsymbol{\eta}(t-1) + \boldsymbol{W}\boldsymbol{x}(t-1), \tag{1}$$

$$\boldsymbol{\zeta}(t) = k_r \boldsymbol{\zeta}(t-1) - \alpha \boldsymbol{x}(t-1) + \boldsymbol{a}, \tag{2}$$

$$\boldsymbol{x}(t) = f(\boldsymbol{\eta}(t) + \boldsymbol{\zeta}(t)), \tag{3}$$

where $\boldsymbol{\eta}$ is a vector representing the internal feedback state of neurons, $\boldsymbol{\zeta}$ denotes the refractory term, $\alpha$ is the scaling parameter of refractoriness, $\boldsymbol{x}$ represents the neuronal state, and $\boldsymbol{W}$ is the connection weight matrix between neurons. The output function adopts a sigmoid form, denoted as $\boldsymbol{f}(\boldsymbol{x}) = (1 + exp(-\boldsymbol{x}/\varepsilon))^{-1}$. The parameters $k_r$ and $k_f$ are the decay parameters of the refractory state and the internal feedback state, respectively, and satisfy the corresponding constraints $0 < k_r, k_f < 1$. $\boldsymbol{a}$ is also a vector that includes the external inputs and corresponding activation threshold values of the neurons. When the connection weight matrix $\boldsymbol{W}$ is determined with the Hebbian learning rule, the network shows strong associative memory capability.

For an $M \times M$ lattice neural network, the number of neurons is $N = M^2$. In a neural network composed of $N$ neurons, the inter-neuronal connections usually adopt either global connection or local connection. Considering that the number of near neighbor connections in nervous systems is much larger than that of long-range connections, local connection between neurons is adopted in this work. Periodic boundary conditions are applied in the network. For a locally conneced network, a semi-connection distance $d$ is defined. For the neuron $i$, the set of neurons directly connected to it is denoted as $S_i$, which contains $(2d+1)^2$ neurons. Let $dL$ and $dT$ denote the longitudinal and transverse distances between two neurons, respectively. Neurons $i$ and $j$ are considered to be connected if both $dL$ and $dT$ are no greater than $d$. Accordingly, the set of neurons effectively connected to neuron $i$ is denoted as $S_i = \{j | dT(i,j), dL(i,j) \leq d\}$. The connection weights of the locally connected network are defined using the Hebbian learning rule:

$$w_{ij} = \begin{cases} ((2e_i - 1)(2e_j - 1) = & \\ \quad (-1)^{i+j+\lceil \frac{i}{m} \rceil + \lceil \frac{j}{m} \rceil}, & \text{for } j \in S_i (i \in S_j), \\ \quad 0, & otherwise, \end{cases} \tag{4}$$

where$\lceil \cdot \rceil$denotes the ceiling function. In our study, $M = 100$, corresponding to a neural network composed of $100 \times 100 = 10,000$ neurons. The stored pattern $\boldsymbol{e}$ and its inverted pattern $\bar{\boldsymbol{e}}$ are shown in Fig. 1. Neurons with output value 1 are represented by black pixels "■", whereas neurons with output value 0 are represented by white pixels "□". To visualize the network state more intuitively, the neuronal outputs are transformed by applying an exclusive-or (XOR) operation with respect to the stored pattern $\boldsymbol{e}$, defined as $x_i^* = e_i \bigoplus \lceil x_i - 0.5 \rceil$. After the XOR transformation, the stored pattern $\boldsymbol{e}$, the inverted pattern $\bar{e}$, and a representative network output shown in the first row of Fig. 1 are converted into the corresponding patterns shown in the second row of Fig. 1.

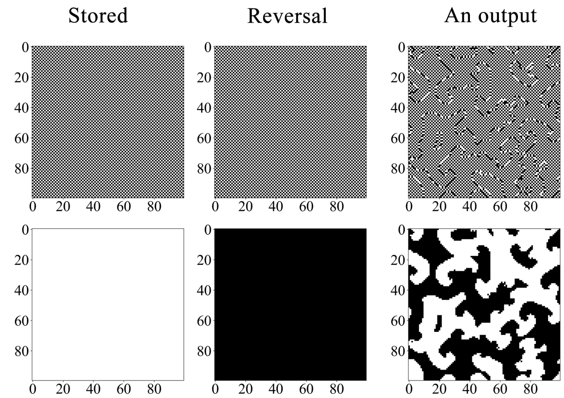


Fig. 1. The stored pattern $\boldsymbol{e}$, its inverted pattern $\bar{\boldsymbol{e}}$, and a representative network output pattern (top), together with their corresponding patterns after the XOR transformation (bottom).

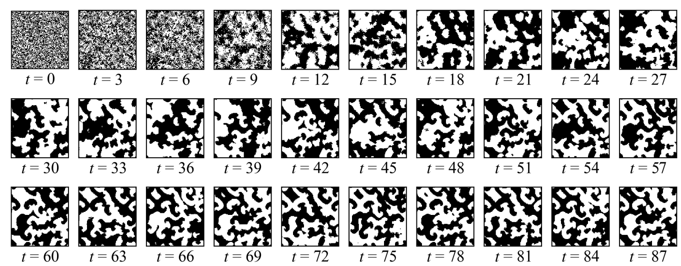


Fig. 2. Spiral wave formation process.

### B. The formation of spiral waves

For the system described by Eqs. (1)–(3), the initial conditions are set as follows. At the initial time ($t = 0$), the refractory variable is initialized as $\zeta_i(0) \in [-\frac{\alpha}{2(1-k_r)}, +\frac{\alpha}{2(1-k_r)}]$, $\eta_i(0) \in [-\frac{(2d+1)^2}{2(1-k_f)}, +\frac{(2d+1)^2}{2(1-k_f)}]$ are randomly selected, whereas $x_i(0) \in [0, 1]$ is assigned with 0 or 1 randomly, satisfying $\Sigma_{(i=1)}^{n} x_i \approx 5000$. These rules are stipulated to avoid extreme scenarios in which neuron clusters exhibit all dormancy or all activation. The network parameters are set to $d = 2$, $k_r = 0.95$, $k_f = 0.15$, $\alpha = 4.0$, and $\varepsilon = 0.02$. The net external input a to all neurons in the network are uniform with $\alpha/2$. Fig. 2 shows the formation process of a spiral wave under the given parameter setting. Due to the local connection and associative memory properties of the network, neurons tend to cluster into the stored pattern or its inverted pattern, and the two patterns coexist and compete in space. Starting from a random initial state, local clusters first emerge, followed by the merging of same-pattern clusters into larger structures. A curved interface forms between the two patterns. In regions with large interface curvature, the two patterns rotate around each other, generating spiral cores and ultimately spiral waves.

## III. Results

This section investigates the formation of spiral waves in a locally connected neural network with two dimension. We focus on the effects of the local connection range, refractoriness-

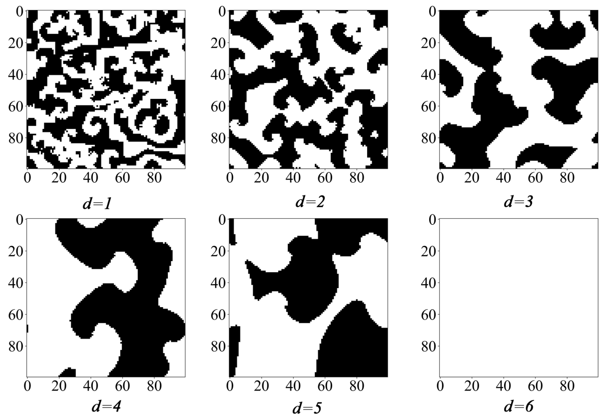

Fig. 3. Effects of different connection ranges $d$.

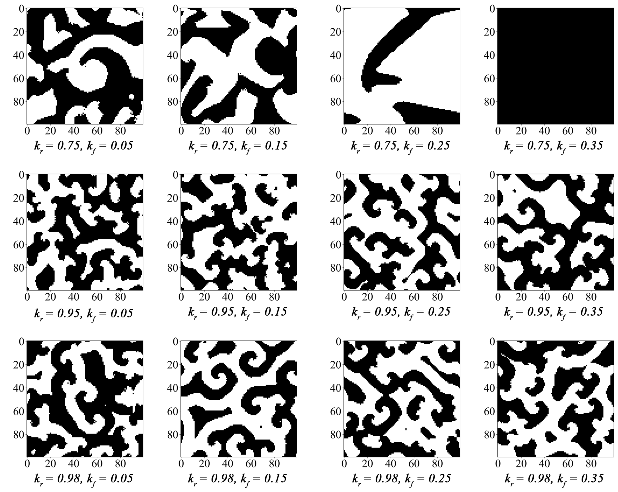

Fig. 4. Spiral wave formation under different $k_f$ and $k_r$ conditions.

related parameters, network size, and boundary conditions on the spatiotemporal dynamics of the network. Unless otherwise specified, all results presented below are obtained under the same initial conditions and parameter settings described in Sect. II.

### A. Effect of local connection range on spiral wave formation

We first examine the influence of the semi-connection distance on the spatiotemporal dynamics of the network. By changing the local connection radius $d$, the generation and evolution of spiral waves under different connection ranges are systematically investigated. In the simulations, $d = 1, 2, 3, 4, 5, 6$ are considered. Numerical results show that for the network with its size of $M = 100$, clear spiral wave structures can be observed when $d$ is taken from 1 to 5. As the connection radius $d$ increases, the spatial morphology of the spiral waves changes significantly. For smaller $d$, spiral waves appear more fragmented, with thinner arms and larger curvature. With $d$ increasing, the spiral waves become smoother, the arms widen, and the overall curvature decreases. When the conection range is further increased to $d \geq 6$, spiral waves gradually disappear and the network tends toward a globally synchronized state, with greatly simplified spatial structures. This indicates that an excessively large local conncetion range suppresses spatial heterogeneity, which is unfavorable for the maintenance of complex spatiotemporal patterns such as spiral waves. Overall, the local connection range is a key structural parameter affecting the generation and stability of spiral waves. A relatively small connection range helps preserve spatial gradients and local asynchrony, providing favorable conditions for spiral wave formation, whereas an overly large connection range, corresponding to strong spatial connection, tends to drive the system into a synchronized dynamical regime, thereby hindering the generation and propagation of spiral waves.

### B. Effects of $k_r$ and $k_f$ on spiral wave formation

$k_r$ is the decay parameter of the refractory term. Refractoriness refers to the reduced or lost responsiveness of a neuron to subsequent stimuli for a period after excitation, reflecting the recovery rate of a neuron from its previous firing. $k_f$ is the decay parameter of the feedback term, which is associated with interactions among neurons through feedback from other neurons. $k_f$ modulates the rate at which feedback effects decay over time. We investigate spiral wave formation by selecting $k_f = 0.05, 0.15, 0.25, 0.35$ and $k_r = 0.75, 0.95, 0.98$. The results show that spiral waves can be generated under most parameter combinations, except $k_r = 0.75$, where spiral wave formation becomes difficult.

### C. Effect of $\alpha$ on spiral wave formation.

$\alpha$ is a scaling parameter that regulates the strength of refractoriness, directly determining the extent to which a neuron is influenced by its past firing activity between successive time steps. In this study, $\alpha = 1, 2, 3, 4, 5, 6$ are selected to systematically inspect the spatial morphology and stability of spiral waves under different $\alpha$ values, as shown in Fig. 5. The results show that for small $\alpha$, the spiral waves formed in the network exhibit a larger spatial scale, with wider spiral arms and a smoother overall structure. As $\alpha$ increases, the spatial scale of the spiral waves decreases, the spiral arms become thinner, and the overall curvature increases. In conclusion, for larger $\alpha$, spiral waves can still stably exist, but is confined to more localized spatial regions.

### D. Effect of network size $M$ on spiral wave formation

To examine the effect of network size, numerical simulations are performed on two-dimensional networks with different sizes. The values $M = 20, 50, 100, 200, 500$ are considered.

The results show that when the network size is small (e.g., $M = 20$), the limited spatial extent causes the spiral core and its arms to be strongly affected by the boundaries, making it difficult for spiral structures to fully develop and leading to distortion or instability. As the network size increases ($M \geq 50$), more complete spiral wave structures can gradually form in the network. When the network size is further increased to $M = 100, 200, 500$, as long as the spiral core and its arms are fully

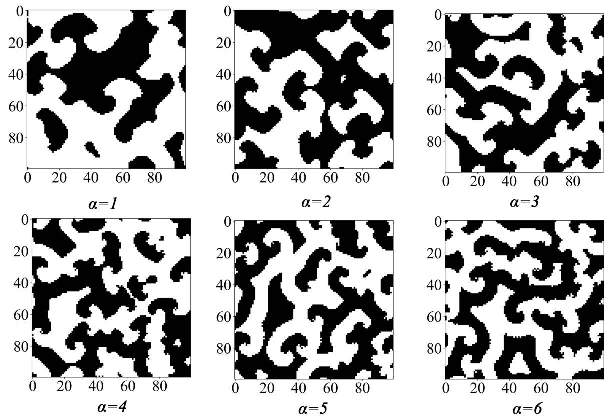


Fig. 5. Effects of different $\alpha$ values on spiral wave formation.

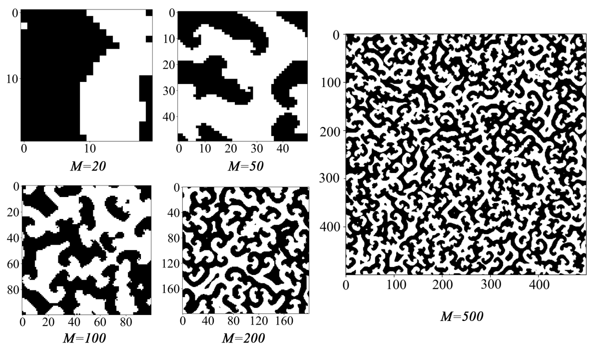


Fig. 6. Effects of different network sizes on spiral wave formation.

contained within the interior of the network, the conditions for spiral wave generation, spatial morphology, and overall dynamical characteristics remain essentially unchanged. The main differences among networks of different sizes lie in the number of spiral waves that can be accommodated and their spatial distribution, while the local structure of an individual spiral wave exhibits good scale invariance.

### E. Boundary conditions: open boundary

In addition to network structure and neuronal dynamical parameters, boundary conditions also play an important role in shaping spatiotemporal dynamics. For an $M \times M$ lattice neural network, two types of boundary conditions are considered. One is the periodic boundary condition, in which the left and right boundaries as well as the top and bottom boundaries are connected, allowing direct information transmission across boundaries. The other is the open boundary condition, in which the left–right and top–bottom boundaries are disconnected, and neurons at the boundaries do not directly exchange information.

In the previous results, periodic boundary conditions are mainly used to investigate spiral wave formation and its parameter dependence. To further examine the effect of boundary conditions on spiral wave dynamics, the boundary condition is switched from periodic to open while keeping the network structure and neuronal parameters unchanged, and the system evolution is compared. The results show that at the early stage

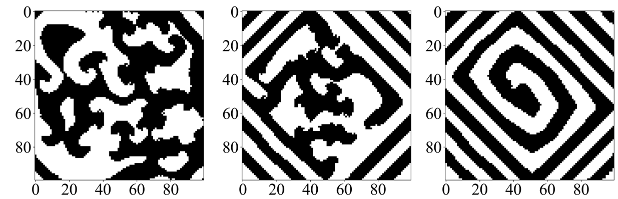

Fig. 7. Evolution of spiral waves under open boundary conditions

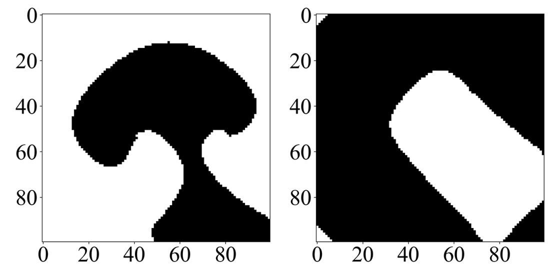

Fig. 8. Direct generation of spiral waves or target waves in an open-boundary network with $d = 5$.

of evolution, spiral waves generated under open boundary conditions do not exhibit significant differences in overall morphology compared with those under periodic boundary conditions, and clear spiral structures can still form in the interior region of the network. However, as time progresses, neurons at the boundaries, due to the reduced number of effective connections, gradually deviate in their dynamics from those in the interior, producing cumulative effects on neighboring regions. After sufficient evolution, spiral waves influenced by boundary effects emerge, as shown in Fig. 7.

When $d$ takes larger values, the number of connections of boundary neurons increases, and boundary effects are partially alleviated. As a result, the interior region of the network can still maintain relatively stable spiral wave structures, which are similar to those generated under periodic boundary conditions with the same parameter settings.

## IV. Discussion and conclusion

Recent experimental studies have shown that spiral waves are not rare or pathological dynamical phenomena, but rather a fundamental form of spatiotemporal organization widely observed across different brain states. Xu et al. reported that interacting spiral waves identified from large-scale fMRI data can organize complex brain dynamics and participate in the dynamic reconfiguration of cognitive processes [3]. Furthermore, analyses of high-density EEG recordings during sleep revealed that spiral waves are significantly associated with memory consolidation, and that their stability and reproducibility can predict individual memory retention performance [19].

This work systematically reveals that the emergence of spiral waves in locally connected chaotic neural networks relies on the synergistic interplay among refractoriness regulation, feedback time scales, and spatial structures. In contrast to previous studies mainly based on continuous reaction–diffusion equations or continuous-time neuronal models, we adopt a discrete-time Aihara chaotic neuron model and systematically investigate the effects of network parameters and boundary conditions on spiral wave formation in strictly locally connected two-dimensional networks. The results demonstrate that, without introducing explicit spatial continuity, spiral waves do not require fine-tuned conditions; instead, the nonequilibrium dynamics arising from the intrinsic refractory memory term and local connection are sufficient to enable the network to self-organize into spiral waves with well-defined phase singularities.

The interface of a spiral wave corresponds to the boundary between the patterns $\boldsymbol{e}$ and $\bar{\boldsymbol{e}}$. In the $\zeta - \eta$ phase plane, this interface is represented by the condition $\zeta + \eta = 0$. A pattern transition indicates that a neuron satisfies $(\zeta^0 + \eta^0)(\zeta^1 + \eta^1) < 0$ , at two consecutive time steps of $t_0$ and $t_1$. By approximating $2f(y) - 1$ with the sign function $sgn(y)$, we obtain

$$(\zeta^0 + \eta^0)(k_r\zeta^0 + k_f\eta^0 - \frac{1}{2}\alpha\left(1 + sgn(\zeta^0 + \eta^0)\right) + a + (\boldsymbol{W}\boldsymbol{x}^0)_i) < 0. \tag{5}$$

Based on the definition of the weight matrix $\boldsymbol{W}$, $(\boldsymbol{W}\boldsymbol{x}^0)_i$ can be written as:

$$\begin{aligned}(\boldsymbol{W}\boldsymbol{x}^0)_i &= \sum_{j=1}^{n} w_{ij}x_j^0 = (2e_i - 1)\sum_{j\in S_i}\left(e_j - |e_j - x_j^0|\right) \\ &= [F_i - H_i(\boldsymbol{x}^0)](2e_i - 1),\end{aligned} \tag{6}$$

where $F_i = \sum_{j\in S_i} e_j$ denotes the number of excited (active) neurons within the connection range of neuron $i$ in the stored pattern $\boldsymbol{e}$, and $H_i(\boldsymbol{x}^0) = \sum_{j\in S_i}|e_j - x_j^0|$ is the generalized Hamming distance between $\boldsymbol{e}$ and $\boldsymbol{x}^0$ within the connection range of neuron $i$ at time $t_0$. To satisfy the above spiral wave propagation condition, the generalized Hamming distance must satisfy:

$$H_i(\boldsymbol{x}^0) < F_i + \frac{1}{2}\alpha + (k_r - k_f)\,\zeta', \tag{7}$$

$$H_i(\boldsymbol{x}^0) > F_i - \frac{1}{2}\alpha - (k_r - k_f)\,\zeta', \tag{8}$$

where $\zeta'(> 0)$ indicates that $|\zeta^0|$ is approximately constant. This constitutes the condition required for spiral wave propagation.

When the connection range $d$ is varied, $F_i$ changes, whereas $\frac{1}{2}\alpha + (k_r - k_f)\zeta'$ remains unchanged. For small $d$, $F_i$ is small and the quantity $F_i + \frac{1}{2}\alpha + (k_r - k_f)\,\zeta'$ becomes relatively large, making the propagation condition easier to satisfy. As a result, spiral waves exhibit the largest curvature at $d = 1$. As $d$ increases, $F_i + \frac{1}{2}\alpha + (k_r - k_f)\,\zeta'$ becomes approximately equal to $F_i = (|S_i| \pm 1)/2$, and it becomes difficult for $H_i(\boldsymbol{x}^0)$ to simultaneously satisfy Eqs.7 and 8. Consequently, the condition for spiral wave propagation can no longer be met. Consistent with in vivo cortical observations reported by Huang et al. [1], our results demonstrate that local interactions play a crucial role in spiral wave formation. When the local connection range becomes too large, the network tends toward global synchronization and spiral waves disappear, which is mechanistically consistent with experimental findings that enhanced long-range connections suppress spiral waves.

The effect of the parameter $\alpha$ can also be analyzed using Eqs.7 and 8. When $\alpha$ is small, both $\alpha$ and $\zeta'$ are small, making it difficult for $H_i(\boldsymbol{x}^0)$ to satisfy the spiral wave propagation condition. As $\alpha$ increases, the range of $H_i$ that satisfies the propagation condition expands. Since $\alpha$ scales the strength of refractoriness, weak refractoriness allows neurons to be reactivated almost immediately after firing, causing local regions to rapidly synchronize and preventing spiral wave formation at small spatial scales. With stronger refractoriness, neurons remain suppressed for a period after firing, introducing clear temporal differences between neighboring regions and enabling excitation to propagate spatially. As a result, with increasing $\alpha$, the spatial scale of spiral waves decreases accordingly.

Varying $k_r$ and $k_f$ requires particular attention to their difference $k_r - k_f$. When this difference is small, the term $(k_r - k_f)\zeta'$ is also small, making the propagation condition more difficult to satisfy. Here, $k_r$ is the decay parameter of the refractory term, while $k_f$ is the decay parameter of the feedback term. This indicates that the relative decay parameters between refractoriness and feedback, rather than either mechanism alone, are the key factors determining spiral wave formation.

The introduction of open boundary conditions reduces the number of connected neurons at the boundaries, so that $(\boldsymbol{W}\boldsymbol{x})_i$ at the boundary is smaller than that in the interior of the network. The reduced neighborhood size $S_i$ leads to smaller values of the feedback term $\eta$, particularly at the four corners, and this effect gradually influences the entire network. Only when $d$ is relatively large does the increase in $S_i$ partially compensate for this effect, making the boundary influence less pronounced. This boundary-induced weakening of feedback introduces intrinsic spatial heterogeneity, causing boundary neurons to detach from global synchronization earlier or more easily. As the system evolves, such local differences can propagate inward through network connections, affecting the spatiotemporal dynamics of the entire network. This process provides initial perturbations for spiral wave generation without relying on external noise or specially designed initial conditions, offering an alternative mechanism for spiral wave formation.

Taken together, the above results indicate that spiral wave formation is not triggered by a single parameter, but is jointly determined by the strength of refractoriness, the time-scale difference between refractoriness and feedback, and the spatial connection structures. These factors collectively regulate the local excitability of neurons and the continuity of spatial phase, thereby shaping propagating excitation patterns in the network.

Previous study by Xu et al. has shown that the spatiotemporal stability of spiral waves is closely related to memory functions [3]. The neural network adopted in this work intrinsically possesses associative memory capability. Recent experimental studies further suggest that spiral waves emerging during sleep are strongly correlated with memory consolidation [18].

Although specific cognitive tasks are not explicitly modeled here, our results imply that spontaneously generated spiral waves in networks with memory storage ability may provide a dynamical substrate for the spatiotemporal organization of memory-related information.

In summary, we employ a neural network to simulate the generation of spiral waves in nervous systems and show that their emergence and propagation depend on network parameters and structures, provided that the corresponding propagation conditions are satisfied. From a broader perspective, traveling wave patterns such as spiral waves are increasingly recognized as being closely associated with information transmission and memory-related processes in the brain. The present results offer a simplified yet controllable modeling framework for understanding spatiotemporal activity patterns in finite-size, structurally heterogeneous nervous systems. A deeper understanding of the conditions for spiral wave generation and their dynamical behaviors may contribute to elucidating memory-related mechanisms in the brain, and such insights into brain dynamics may, in turn, inspire future developments in artificial intelligence.


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