Global Sensitivity Analysis of Models with Correlated Inputs

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Abstract

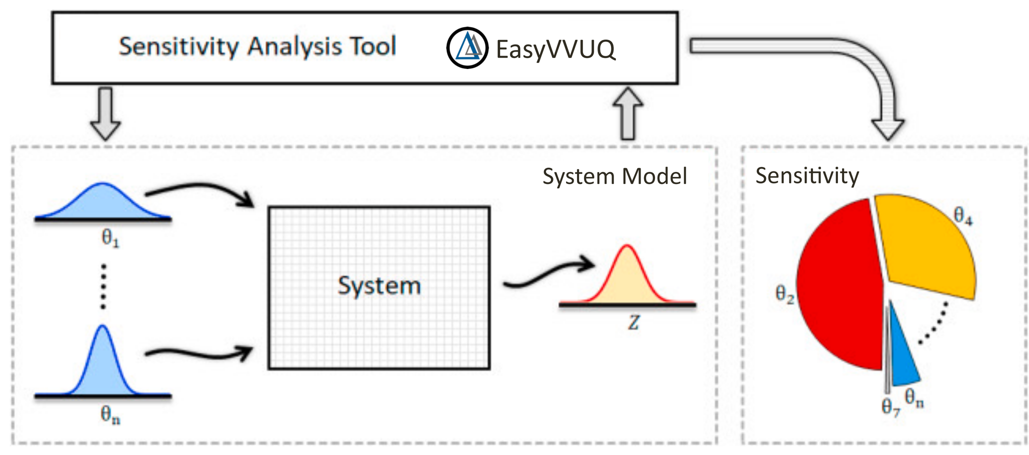
Global sensitivity analysis is an important tool used in many domains of computational science to either gain insight into the mathematical model and interaction of its parameters or study the uncertainty propagation through the input-output interactions. This works introduces a comprehensive **framework for conducting global sensitivity analysis on models with correlated inputs**. Traditional sensitivity analysis methods assume independence between inputs and can provide misleading results when this assumption is violated. The proposed approach addresses parameter correlations using Rosenblatt transformation, which are incorporated into a polynomial surrogate model. The sensitivity analysis requires numerous execution of the target application, which requires significant computational resources. The numerical experiments are executed using HPC platforms equipped with metaschedulers and workflow automation tools.

Why Sensitivity Analysis

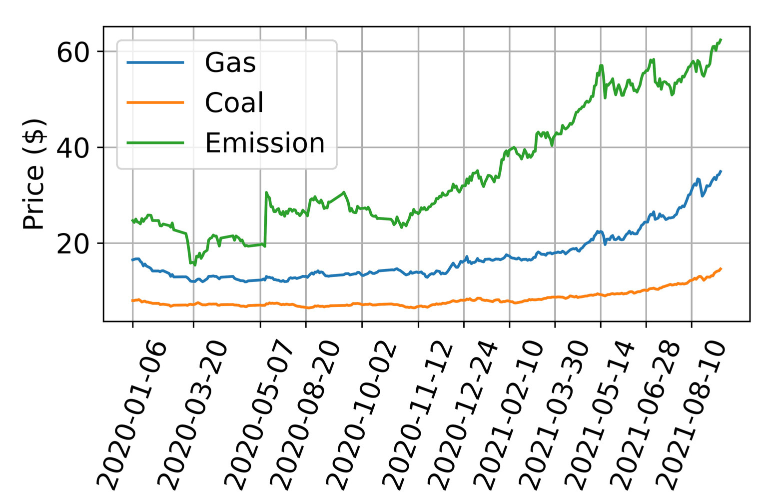
SA is now considered **a requirement for good modelling** (e.g. *"Better regulation toolbox"* by European Commission, 2015).

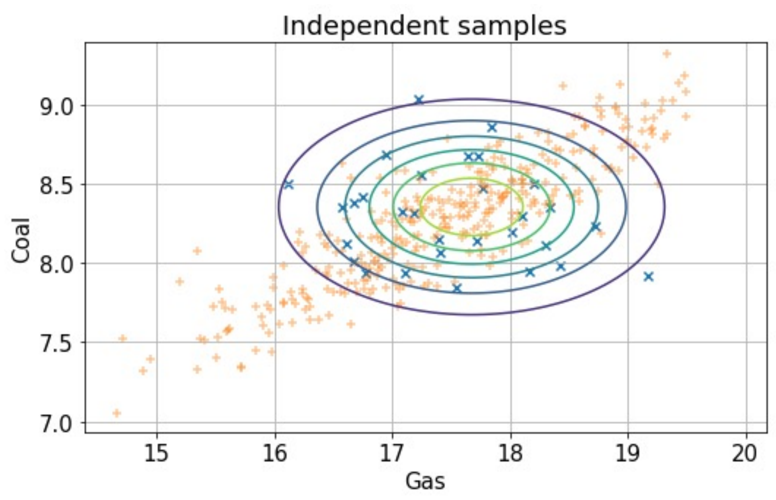
SA helps to get insights about:

- How the **outputs** of a system are related to, and are influenced by, its **inputs**
- Identify **uninfluential factors** in a system that may be redundant and fixed or removed
- Identify parameters that dominantly control a system, for which new data acquisition reduces uncertainty the most
- Quantify the sensitivity of an expected outcome to different decision options, constraints, assumptions and/or uncertainties



Sensitivity Analysis

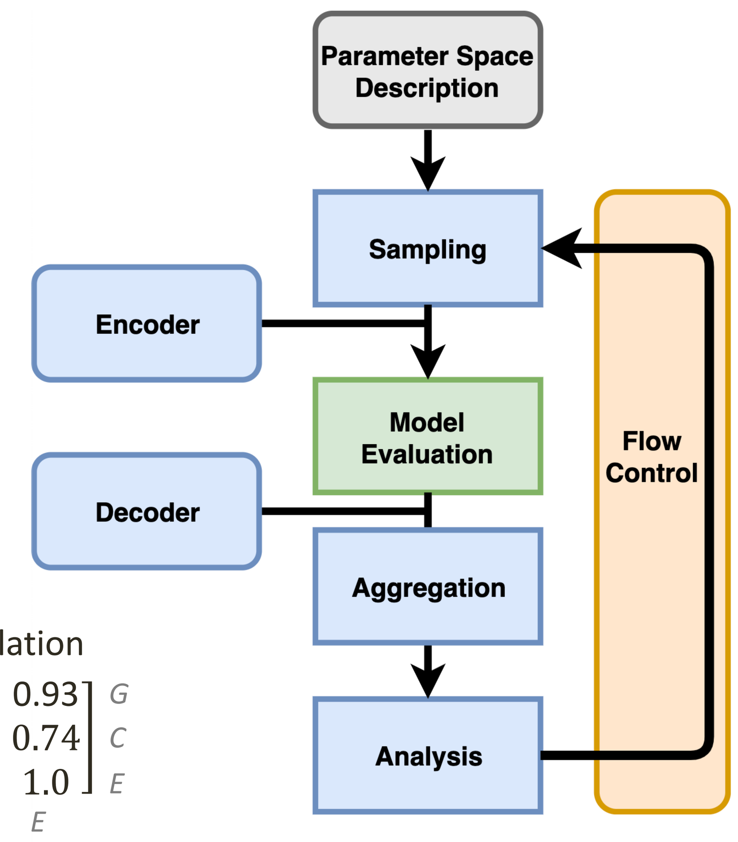




In many applications, the **inputs are stochastically dependent**, which violates one of the essential assumptions in the state-of-the-art sensitivity analysis methods. Consequently, the SA results obtained ignoring the correlations provide values which do not reflect the true contributions of the input parameters.

Inputs Correlation

	G	C	E
G	1.0	0.82	0.93
C	0.82	1.0	0.74
E	0.93	0.74	1.0



Derivative-based approach $S_i^{\mathcal{D}} = \left.\frac{\partial Y_n}{\partial Q_i}\right|_{Q_i^0}.$

Variance-based approach $S_i = \frac{V_i}{V(Y_n)}.\quad V_i = \mathbb{V}(\mathbb{E}(Y_n|Q_i)),$

$$V(Y_n) = \sum_i V_i + \sum_i \sum_{j>i} V_{ij} + \ldots + V_{12\ldots D},$$

Algorithm 2 SA Method with Correlations.

Generation of samples and their transformation:

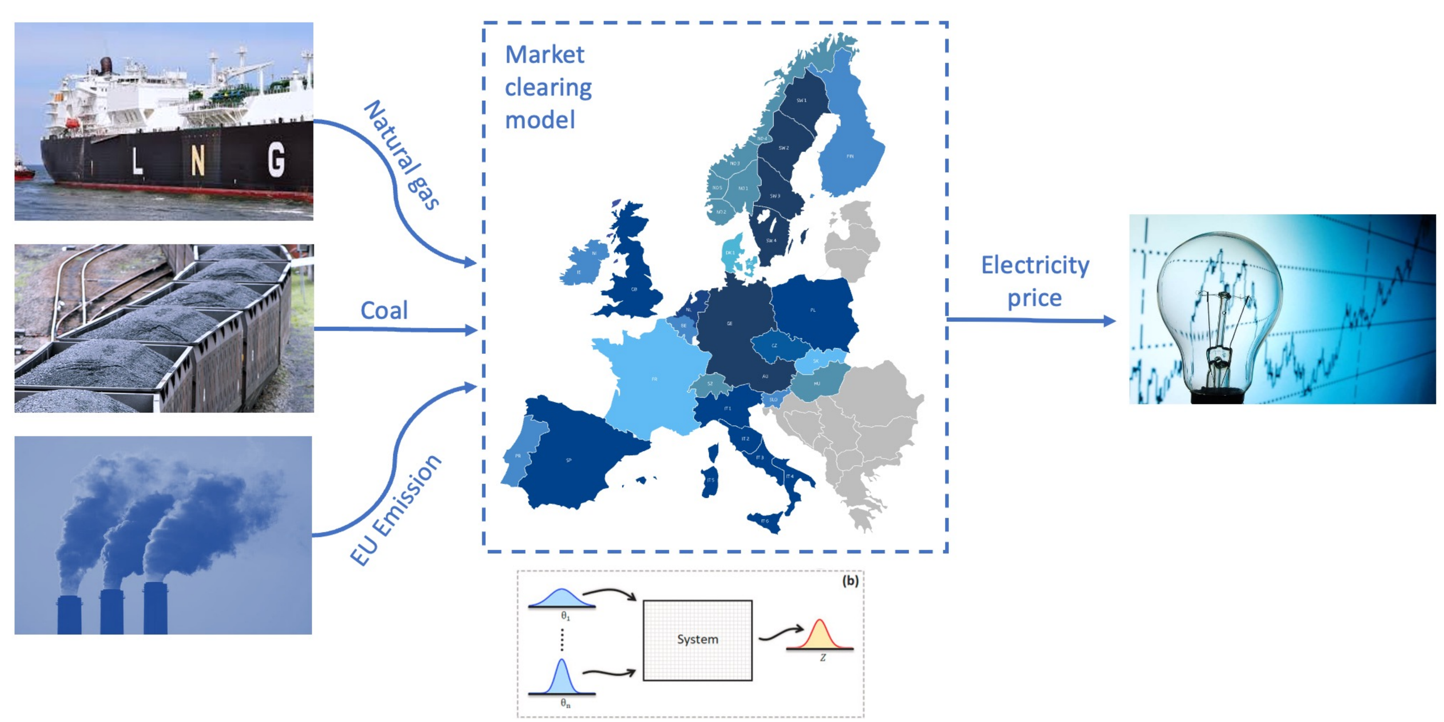
1. Generate samples $\mathbf{q}_1, \ldots, \mathbf{q}_N$ from the independent multivariate distribution $\rho_{\boldsymbol{Q}}$.
2. Transform the samples $\mathbf{q}_i \in \rho_{\boldsymbol{Q}}$ to $\mathbf{q}_i^* \in \rho_{\boldsymbol{Q}}^*$, $i = 1, \ldots, N$, using:
   (a) Cholesky transformation $\mathbf{q}_i^* = T(\mathbf{q}_i) = \mathbf{q}_i L$ from (14).
   (b) Rosenblatt transformation $\mathbf{q}_i^* = T(\mathbf{q}_i)$ from (18).

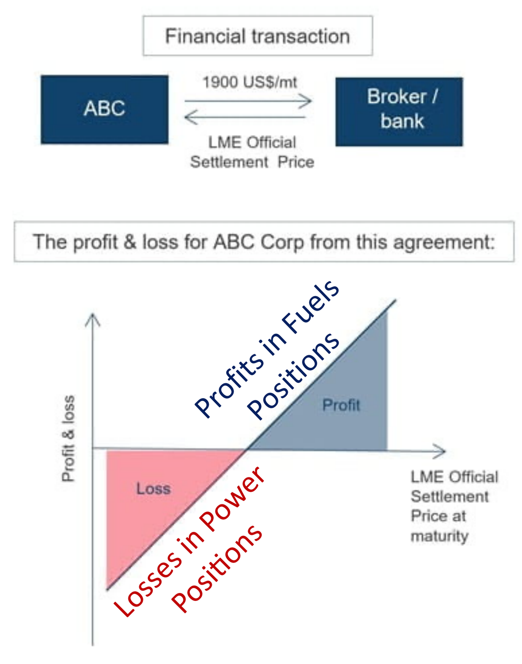
Construction of the surrogate model:

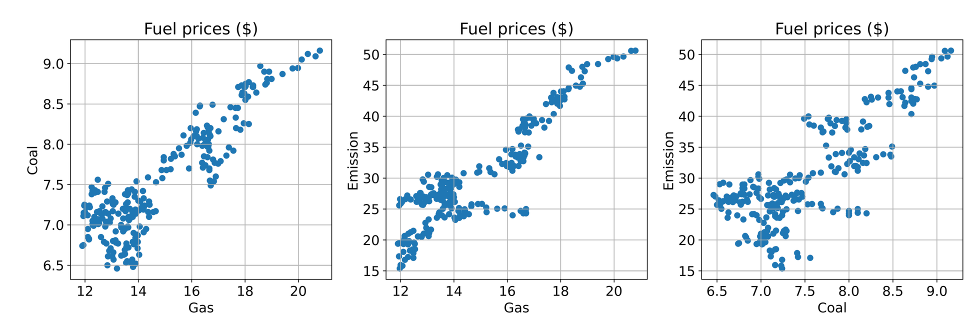
1. Evaluate the true model $\boldsymbol{Y}_1^* = U(\boldsymbol{x}, \mathbf{t}, \mathbf{q}_1^*), \ldots, \boldsymbol{Y}_N^* = U(\boldsymbol{x}, \mathbf{t}, \mathbf{q}_N^*)$ at $\mathbf{q}_i^* \in \rho_{\boldsymbol{Q}}^*$.
2. Create a polynomial expansion $\Psi_1, \ldots, \Psi_P$ up to the P-th degree from $\rho_{\boldsymbol{Q}}$.
3. Solve the linear regression problem: $\boldsymbol{Y}_n^* = \sum_p a_p \Psi_p(\boldsymbol{Q}_n)$ for $a_1, \ldots, a_p$.
4. Construct the model approximation $U(\boldsymbol{x}, \mathbf{t}, \boldsymbol{Q}^*) \approx \hat{U}(\boldsymbol{x}, \mathbf{t}, \boldsymbol{Q}) = \sum_p a_p \Psi_p(\boldsymbol{Q})$.

Application - Delta Hedging

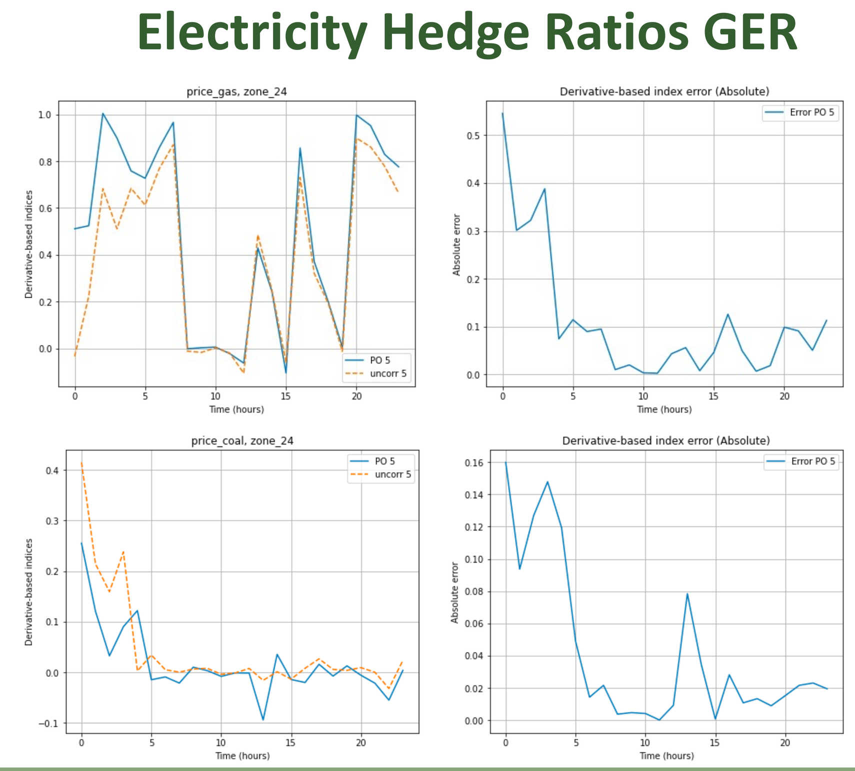
Sensitivity = Hedge Ratios





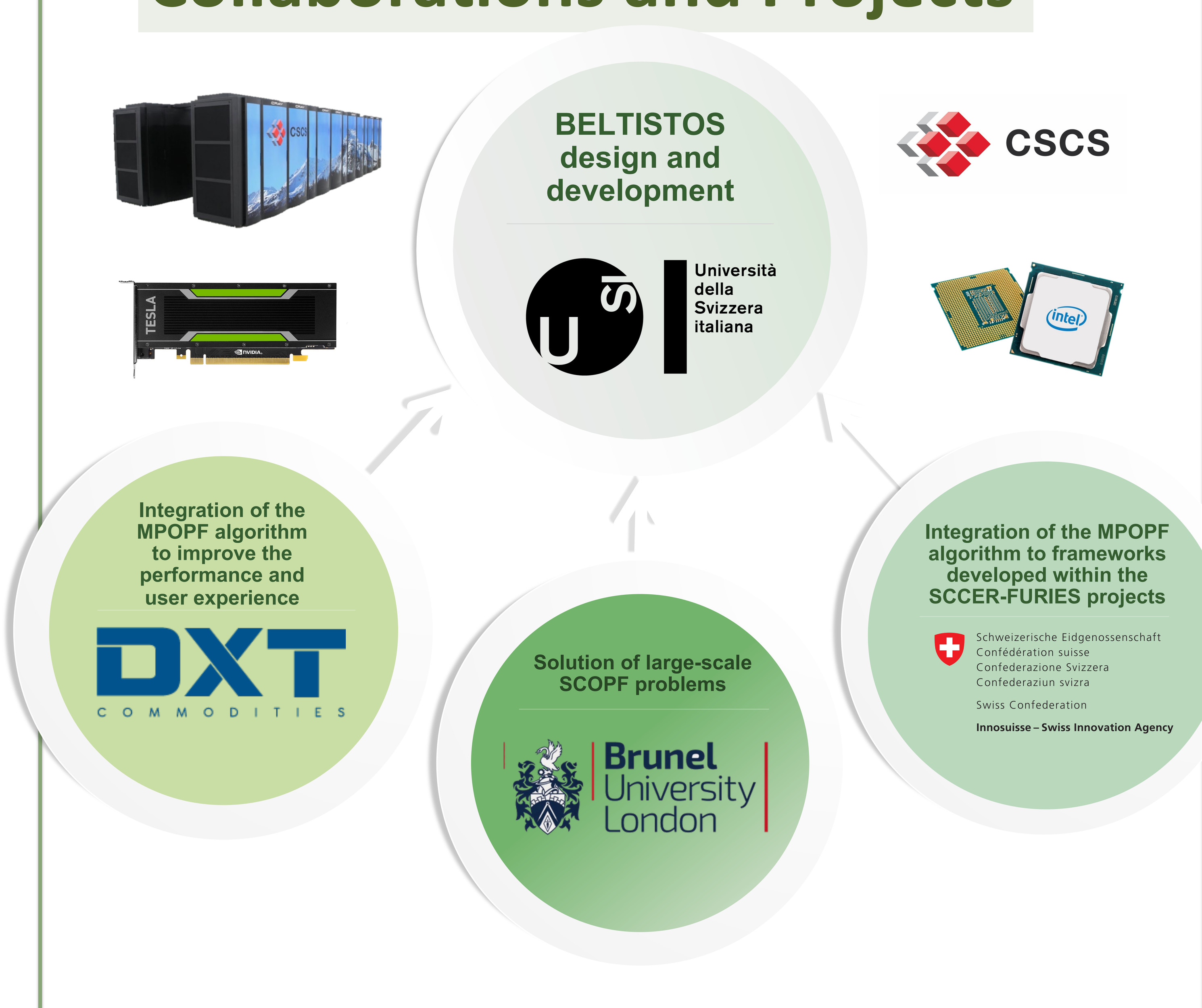


Electricity Hedge Ratios GER



In the presence of parameter correlations, the resulting power **price sensitivity changes** compared to the base case. Note the pronounced impact of the natural gas input and reduced sensitivity of the coal. This impacts the financial instrument such as hedges of power price with respect to the aforementioned fuels.

Collaborations and Projects



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[1] Juraj Kardoš, Timothy Holt, Vincenzo Fazio, Luca Fabietti, Filippo Spazzini, and Olaf Schenk. ***Massively parallel data analytics for smart grid applications***. Sustainable Energy, Grids and Networks, 31:100789, 2022. doi: doi.org/10. 1016/j.segan.2022.100789

[2] Juraj Kardoš, Drosos Kourounis, Olaf Schenk, and Ray Zimmerman. ***BELTISTOS: A robust interior point method for large-scale optimal power flow problems***. Electric Power Systems Research, 212:108613, 2022. doi: doi.org/10. 1016/j.epsr.2022.108613

[3] Juraj Kardoš, Timothy Holt, Olaf Schenk, Vincenzo Fazio, Luca Fabietti, and Filippo Spazzini. **High-performance data analytics techniques for power markets simulation**. In 2021 International Conference on Smart Energy Systems and Technologies (SEST), pages 1–6, 2021. doi: 10.1109/SEST50973.2021.9543110

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