

NektarIR

Towards code-generation of highly efficient finite element kernels for computational fluid dynamics using the MLIR compiler infrastructure



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Aims

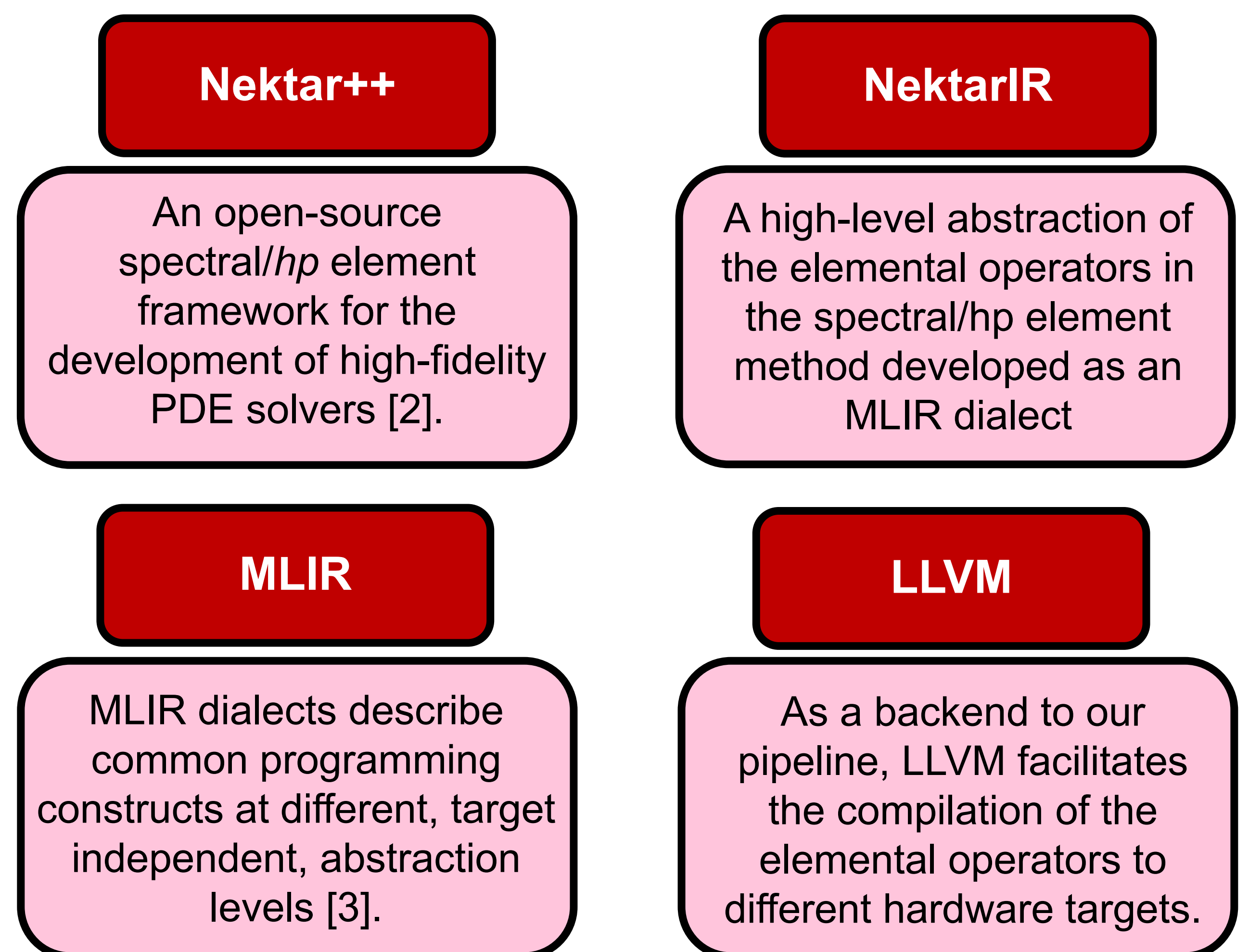
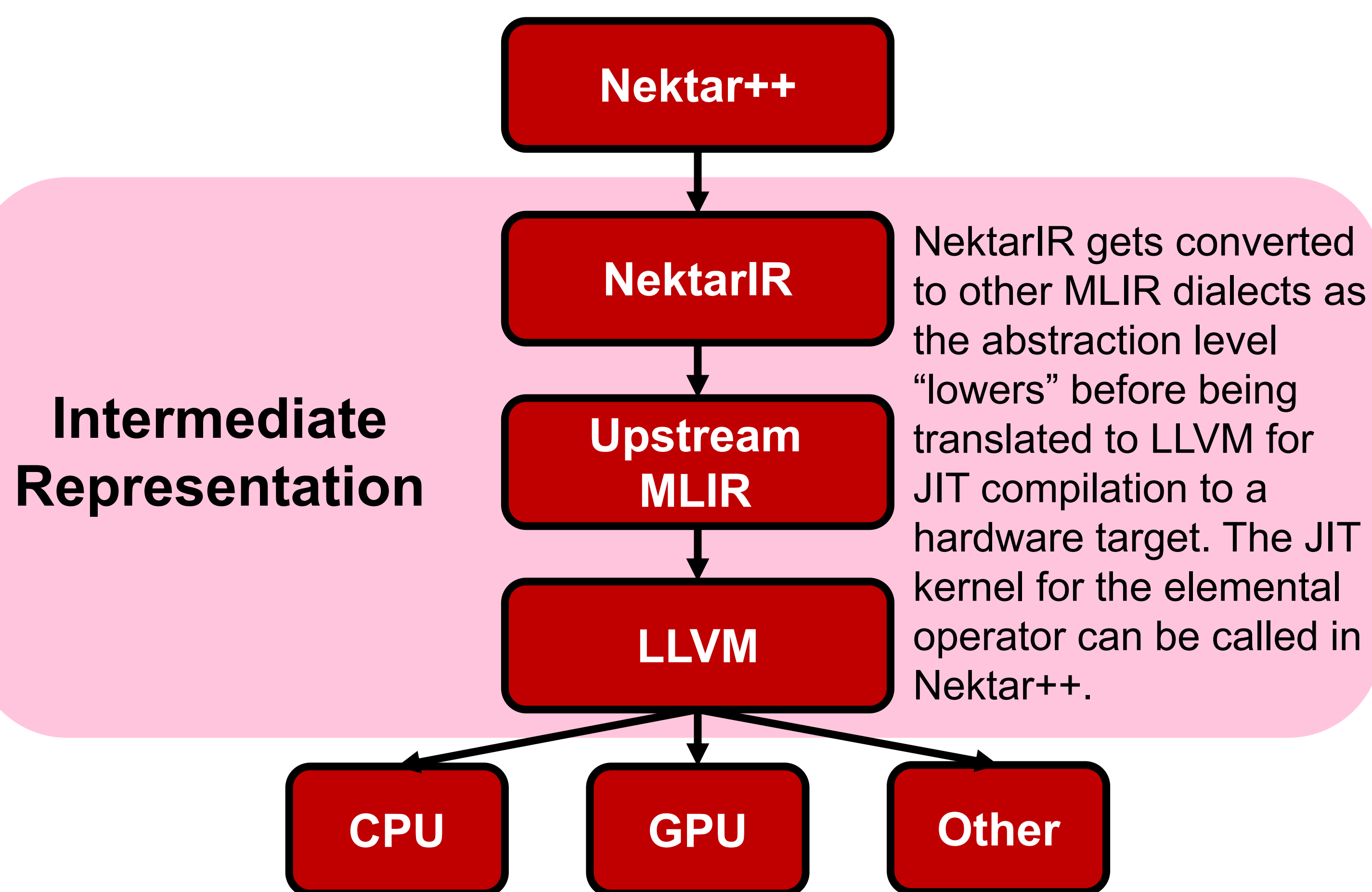
- Create a **high-level abstraction** of the elemental operators for the construction of PDE solvers using the spectral/hp element method
- Separate optimizations from the concerns of domain scientists through **target independent optimizations** using the MLIR compiler development infrastructure
- **Ensure hardware extensibility and facilitate JIT compilation** using LLVM

Spectral/hp Element Method and Elemental Operators

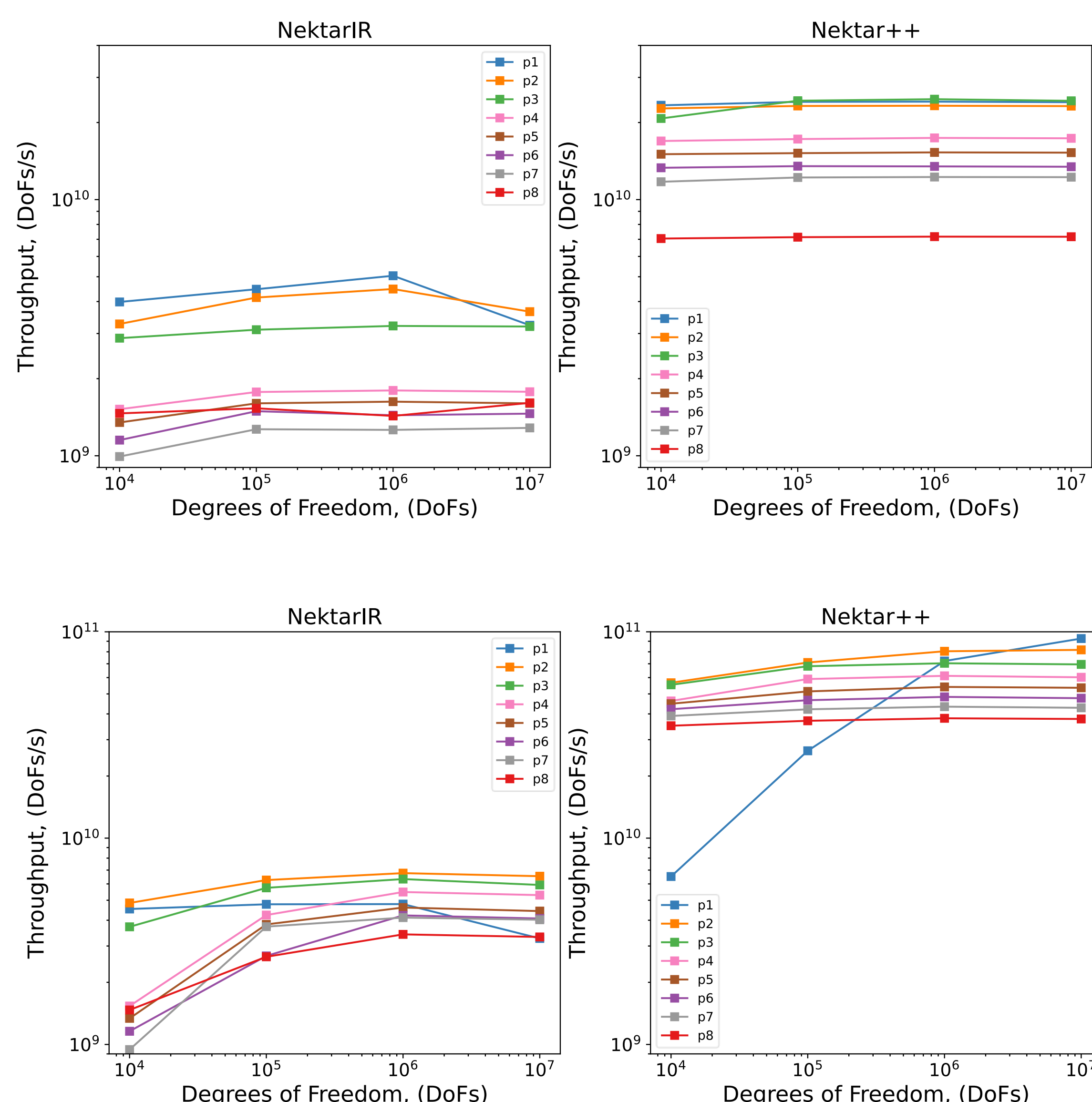
- A **high-order** finite element method for solving partial differential equations used in HPC applications such as computational fluid dynamics
- Implementation requires frequent evaluation of **elemental operators** within elemental regions of a mesh of the solution domain [1], such as:

Backward Transform $u(\xi_{1i}, \xi_{2j}) = \sum_{p=0}^P \sum_{q=0}^Q \hat{u}_{pq} \phi_{pq}(\xi_{1i}, \xi_{2j}) \quad \forall i, j$

Inner product with respect to expansion basis $I_{pq} = \sum_{i=0}^{N_1} \sum_{j=0}^{N_2} u(\xi_{1i}, \xi_{2j}) w_i w_j \phi_{pq}(\xi_{1i}, \xi_{2j}) \quad \forall p, q$



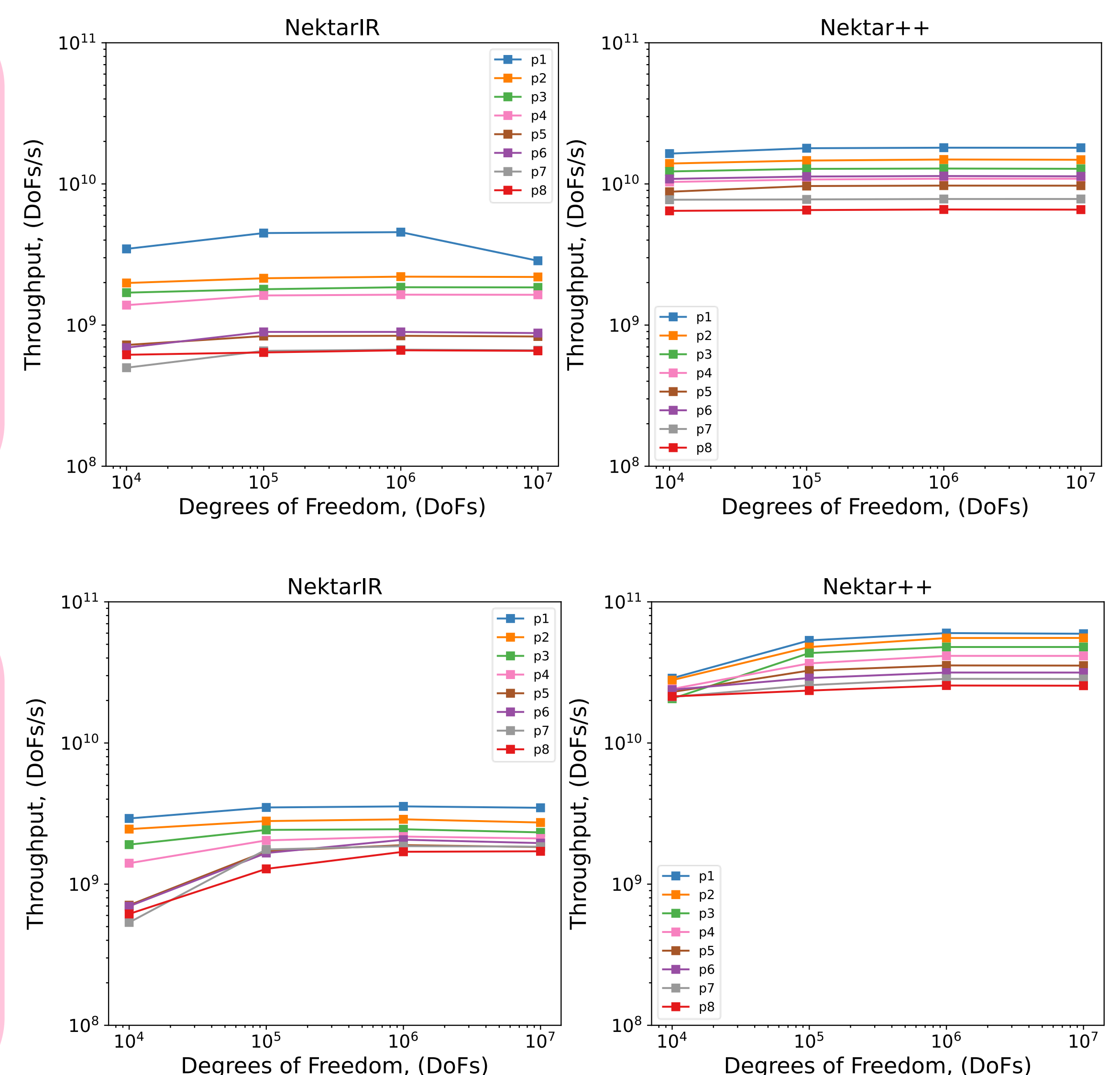
Initial Benchmarking: Backward Transform



Comparison of scalar (top) and vectorized (bottom) throughput performance for two elemental operators on quadrilateral elements. Everything run on 64 cores across two AMD EPYC 9554 CPUs

NektarIR and Nektar++ use different vectorization strategies and results show Nektar++ has better performance. Differences in scalar performance requires further investigation and shows need for optimization.

Initial Benchmarking: Inner Product WRT Basis



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References

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- [2]: C.D. Cantwell, D. Moxey, A. Comerford, A. Bolis, G. Rocco, G. Mengaldo, D. De Grazia, S. Yakovlev, J.-E. Lombard, D. Ekelschot, B. Jordi, H. Xu, Y. Mohamied, C. Eskilsson, B. Nelson, P. Vos, C. Biotto, R.M. Kirby, S.J. Sherwin, Nektar++: An open-source spectral/hp element framework, Computer Physics Communications, Volume 192, 2015, Pages 205-219, ISSN 0010 4655, <https://doi.org/10.1016/j.cpc.2015.02.008>.
- [3]: <https://mlir.llvm.org/>